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Doing it first time at the workplace; vital role of core self-evaluation in the absence of rewards for training transfer

ANWAR UL HAQ (Islamic International University, Islamabad, Pakistan).

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Abstract

This study is an endeavor to empirically investigate the links of resource availability at workplace with transfer of training; an effort has been made to highlight the importance of a relatively newly discovered construct of core self-evaluation (CSE) in this relation besides the traditionally acknowledged trainees motivation to transfer. The study was conducted on 197 trainee techno/managers of energy sector in Pakistan. Hayes' process method was used through SPSS to regress the effects of resource availability factors on transfer of training. The results showed that the work environment factors such as physical and aesthetic environment (PAE), technological support (TS) and budget availability (BS) do have an impact on trainee motivation to transfer (TMT) and in turn on transfer of training (TT); most importantly, positive trainee core self evaluations play the role of an important psychological resource when physical resources demanded by the transfer of training are scant; this moderates the relationship of trainee motivation to transfer and transfer of training positively.

Time Series Analysis and Forecast of India's Wholesale Price Index Inflation

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Abstract

Many central banks use forecasting models based on machine learning methodologies for estimating various macroeconomic indicators, like inflation, GDP Growth and currency in circulation etc. Particularly those central banks that run an inflation targeting regime urgently require high quality inflation forecasts. A sound base for monetary policy decisions requires deep insights into the process that generates future inflation in general and the transmission mechanism from short-term interest rates to long term interest rates, exchange rates, real economic activity and inflation in particular. Inflation forecasting plays a major role in monetary policy and daily life. Based on different diagnostic and evaluation criteria, the best forecasting model for predicting inflation is identified. The results will enable policy makers and businesses to track the performance and stability of key macroeconomic indicators using the forecasted inflation. In this report we have attempted monthly inflation of wholesale price index (WPI) for India by using conventional time series forecasting based ARIMA model and machine learning algorithms on the basis of monthly data between January 2005 to March 2017. The focus of this study is to employ three methods: ARIMA, Artificial Neural Network and Support Vector Machine for forecasting of inflation of WPI. We have carried out inflation forecast for the months till March 2018.

Introduction

Inflation rate is one among the most crucial economic indicators providing an input for monetary policy deliberations. It is also important for financial planning for both enterprises and residents. Without an accurate gauge of the inflation rate, it is difficult to forecast the actual expenses. In a dynamic world, prices do not remain constant. Inflation rate calculated on the basis of the movement of the Wholesale Price Index (WPI) is an important measure to monitor the dynamic movement of prices. The methods used for time series prediction are common to the field of statistics, such as ARMA model, ARIMA model, including various artificial intelligent models, such as artificial neural networks, genetic algorithm and support vector machines. Many central banks including CZECH National Bank(Marek etal, 2005), Bank of Canada (Greg etal, 1999) are using machine learning methods, ANN in particular for estimating various macroeconomic indicators.

In this work a series of monthly forecasts of one macroeconomic aggregate has been carried out. Forecasts have been made using ARIMA model and machine learning algorithms. As these methods require a reasonably long time series data series for WPI has been used that was available from January 2005 to April 2017. Forecasts have been made using much shorter series as it is. Forecasts based on a much shorter time steps are likely to have higher forecast error. The forecasts are given for months till May 2018. There are several approaches available for forecasting economic time series. One among those approach, which includes only the time series being forecast, is known as univariate forecasting. Autoregressive integrated moving average (ARIMA) modelling is a specific subset of this, in which a time series is expressed in terms of past values of itself (the autoregressive component) plus current and lagged values of a 'white noise' error term (the moving average component). In this work ARIMA model has been used as one of the forecast model. Here multistep-ahead prediction is performed which is more complex and troublesome than one step ahead. The investigation that has been carried out proposed that nonlinear relationships may exist among the monthly indices, so that the ARIMA model might not be able to effectively extract the full relationship hidden in the historical data. Hence machine learning algorithm ANN (artificial neural network) and Support Vector Machine has been used for forecasting WPI till May 2018. The reliability of both computational models was analyzed in light of simulation results and it was found out that ANN gives better results with less error as compared to ARIMA.

Keywords—Time Series, Machine learning, ARIMA, Inflation

Time Series Analysis

A time series is a time dependent or chronological sequence of observations on a considered variable. Examples include (i) sales of a particular product in successive months, (ii) the temperature at a particular location at night on successive days, and (iii) electricity consumption in a particular area for successive one-hour periods. However, time-series data presents an excellent opportunity to look at what is called out-of-sample behavior. A time-series model will provide forecasts of new future predictions which can be checked against what is actually observed. If there is good agreement between each other, it will be argued that this provides a more convincing verification of the model than insample fit [1].

ARIMA Model

ARIMA models are considered as the most general class of models for forecasting a time series which can be made to be stationary by differencing (if required), perhaps in conjunction with nonlinear transformations such as logging or deflating (if required). A random variable with time series nature is considered to be stationary if its statistical properties are all consistent over time. A stationary series has no trend, its variations around its mean have a constant amplitude, and it wiggles in a consistent fashion, i.e., its short-term random time patterns always look the same in a statistical sense. The latter condition means that its autocorrelations (correlations with its own prior deviations from the mean) remain constant over time, or equivalently, that its *power spectrum* remains constant over time. A random variable of this form can be viewed (as usual) as a combination of signal and noise, and the signal (if one is apparent) could be a pattern of fast or slow mean reversion, or sinusoidal oscillation, or rapid alternation in sign, and it could also have a seasonal component. An ARIMA model can be viewed as a filter that tries to separate the signal from the noise, and the signal is then extrapolated into the future to obtain forecasts.

The ARIMA forecasting equation for a stationary time series is a linear (i.e., regression-type) equation in which the predictors consist of lags of the dependent variable and/or lags of the forecast errors. That is:

Predicted value of Y = a constant and/or a weighted sum of one or more recent values of Y and/or a weighted sum of one or more recent values of the errors.

If the predictors consist only of lagged values of Y, it is a pure autoregressive (self-regressed) model, which is just a special case of a regression model and which could be fitted with standard regression software. If some of the predictors are lags of the errors, an ARIMA model it is NOT a linear regression model, because there is no way to specify last period's error as an independent variable: the errors must be computed on a period-to-period basis when the model is fitted to the data. From a technical standpoint, the problem with using lagged errors as predictors is that the model's predictions are not linear functions of the coefficients, even though they are linear functions of the past data. So, coefficients in ARIMA models that include lagged errors must be estimated by *nonlinear* optimization methods (hill-climbing) rather than by just solving a system of equations.

The acronym ARIMA stands for Auto-Regressive Integrated Moving Average. Lags of the stationarized series in the forecasting equation are called autoregressive terms, lags of the forecast errors are called moving average terms, and a time series which needs to be differenced to be made stationary is said to be an integrated version of a stationary series. Random-walk and random-trend models, autoregressive models, and exponential smoothing models are all special cases of ARIMA models.

A nonseasonal ARIMA model is classified as an "ARIMA(p,d,q)" model, where:

p is the number of autoregressive terms,

d is the number of nonseasonal differences needed for stationarity, and **q** is the number of lagged forecast errors in the prediction equation.

Artificial neural network

Despite its evocative name, an artificial neural network (ANN) is simply a parameterized non-linear function that can be fitted to data for forecast purposes. The processing units or neurons of an ANN consists of three main components; synaptic weights connecting the nodes, the summation function within the node and the transfer function (see Fig4.2). Synaptic weights are known for their strength which corresponding to the importance of the information coming from each neuron. Which means the information is encoded in these strength-weights. The summation function is used to estimate the total input signal by multiplying with their synaptic weights and summing up all the products.

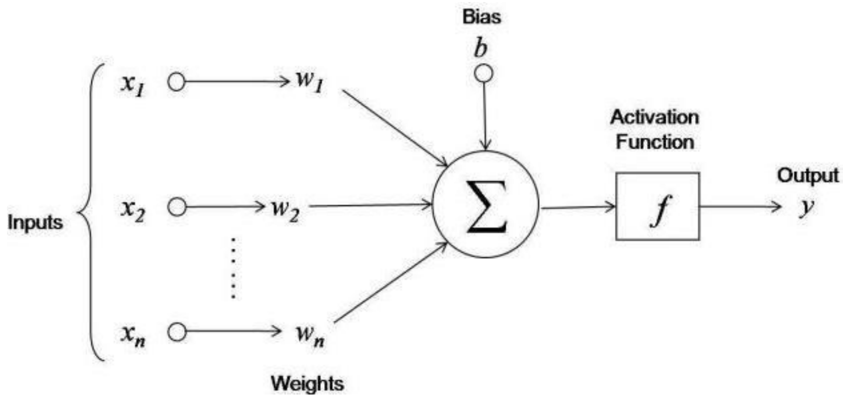

Fig.1. Schematization of artificial neuron

The non-linear function is constructed as a combination of non-linear building blocks, known as transfer functions. A common example of an ANN transfer function is the hyperbolic tangent function. Activation function transforms the summed up input signal, received from the summation function, into an output. The activation function can be either linear or non-linear. The structure of the ANN is described, in neural networks jargon, by the number of neurons and layers in the ANN. These features determine the number and organization of the non-linear transfer functions. Increasing the numbers of transfer functions (adding more neurons and layers) increases the flexibility of the ANN. The main appeal of ANNs is their flexibility in approximating a wide range of functional relationships between inputs and outputs. Indeed, sufficiently complex neural networks are able to approximate arbitrary functions arbitrarily well. Thus, there is a close relationship between the ANN approach and the older economics literature on flexible functional forms such as the translog function. In this study, the neural networks used have the sigmoid function and the linear function as the activation functions of the hidden and output layers, respectively. The dataset used for training, validating, and testing the neural network is divided into three groups. First, the training set, which usually consists of half or more of all data gathered. It is used by the ANN to adjust its weights and biases. Second is the validation set, which is used for validating the network training. It checks the network's capability to generalize a series of input data. Finally, the test set is used to evaluate the network's performance [4]. The latter two sets consist of data that have not been previously presented to the network. The algorithms used to estimate ANNs are known as training algorithms. These algorithms are much like standard minimization routines used, for example, in non-linear least squares. Loosely speaking, the training algorithms iteratively adjust the parameters in the direction of the negative gradient of mean squared error. However, standard practice ANN estimation approaches differ from econometric estimation techniques in important ways. In order to avoid overfitting, the ANN training algorithm is often stopped before a local minimum is reached. Intuitively, overfitting occurs when the ANN provides a near-perfect fit in-sample but poor predictions out-of-sample. ANNs are thought to be particularly susceptible to overfitting because of their flexibility in approximating different functional forms. One of the most common types of early stopping procedures is the following cross-validation-based approach. The training algorithm is run on the training set until the MSE starts to increase on the validation set (which usually occurs long before the minimum MSE is reached on the training set). The errors associated with the stop criteria are the validation (generalization) and training errors. The test error is the network's performance measurement based on the test set. The training, validation, and test errors are evaluated by comparing the actual observed data to the predicted data.

Support Vector Machine

The training number of the SVM based time series model needed to be determined. In previous studies [13], the training number for the training of periodic series is usually the period of the series. In the present study, the period of the entire infectious disease incidence selected is twelve. Therefore, twelve was selected as the training number for SVM based models, in which the last 12 months of data were reserved as the input for forecasting the present data. Proper transition of the data series is always necessary to determine the input and the output data before the training process.

SVM is a learning framework utilizing a high dimensional feature space. It generates the prediction functions that are developed on a subset of support vectors. SVM has the ability to generalize complex structures with help of a very few support vectors hence provides a new mechanism for image compression. This strategy is called support vector regression (SVR). The model generated by support vector classification is only dependent on a subset of the training data, because the cost function for building the model does not care about training points that goes beyond the margin. Similarly, the model generated by SVR only depends on a subset of the training data, on the grounds that the cost function for building the model overlooks any training data that is close (within a threshold ε) to the model prediction. Support Vector Regression (SVR) is the most well-known application type of SVMs. A review of the essential ideas presenting the support vector (SV) regression with function estimation has been presented in [5]. Moreover, it has incorporated an outline of currently used algorithms for training SVMs, covering both the quadratic (or convex) programming part and advanced methods for managing extensive datasets. At last, a few alterations and extensions have been applied to the standard SV algorithm.

Assume we are given training data{$(x_1, y_1), \ldots, (x_l, y_l)$} □□× R, where □means the space of the input patterns (e.g.□= R_d).

In SV regression, the objective has been to find a function $f(x)$ that has at most □ deviation from the actually obtained targets y_i for all the training data at the same time as flat as it can be expected. The case of linear function f has been described in the form as

$f(x)$ □□w,x)□b with w□N,b□R (1)

Where<.,.> presents the dot product in N. Flatness in (1) means small w. For this, it is required to limit the Euclidean norm i.e. $||w||^2$ .Formally this can be represented as a convex optimization problem by requiring

Minimize $\frac{1}{2}||w||^2$

Subject to □□y_i □□w,x_i □□b□□ (2)

□□w,x_i □□y_i □□

The above convex optimization problem is achievable in cases where f actually exists and approximates all pairs (x_i, y_i) with □precision. Once in a while, some errors are allowed. Introducing slack variables □,□$_i$ to cope with otherwise infeasible constraints of the optimization problem (2), the formulation becomes

l $\frac{1}{2}$

Minimize $||w||^2 + C$□(□$_i$ □□$_i$*)

i□1

□y_i □□w,x_i □□b□□□□$_i$

□

Subject to □□w,x_i □□y_i □□□□$_i$* (3)

□□□,□$_i$*□ 0

The constant C >0 decides the trade-off between the flatness of f and the amount up to which deviations larger than ε are tolerated.

There is no structural way to determine the optimal parameters of SVMs. In the present study, cross validation methods were applied to determine the proper SVMs. The training samples were randomly divided into k parts in the training process, each part was used for testing and the others used for training. The obtained MSE each test was recorded and the mean of the MSE acted as the selection criterion for the optimal parameters.

Performance Measures

For the problem of time-series forecasting, there is no single metric universally adopted by researchers to evaluate a model's predictive adequacy. In the present study, the root mean square error (RMSE), which is one of the most common performance measures applied to neural networks is considered to allow a better appreciation of the forecasting system performance. Root Mean Square Error (RMSE) is defined by the standard deviation of the residuals. Residuals (prediction errors) are a representation of how far from the regression line sample points are. RMSE is represented as a measurement of how spread out these residuals are. In other words, it will tell how concentrated the data is around the line of best fit.

n

RMSE□ $^1{}_2$□ (y□y_i)^2 $\sqrt{-}$

i□1

Where y = forecasts (expected values or unknown results), y_i = observed values (known results)

DATA SELECTION

The Ministry of Industry releases its latest Wholesale price Index (WPI) every month. WPI is a weighted average of inflation suitably measured at commodity or group of commodities level.

This report present certain salient features of evolving inflationary dynamics of India and generates forecasts of monthly inflation till May 2017 and compare the results with the actual inflation rates obtained using time series analysis.

Monthly WPI used for estimation were acquired from MOSPI, India. The measurements available for the WPI parameter were collected between the years 2005 and 2017. The dataset consists of 156 averaged monthly observations.

RESULTS AND DISCUSSION

The best ARIMA forecasts for year over year inflation, using WPI data till May 2017 have been presented. We have also used a machine learning algorithm (artificial neural network) for the step ahead WPI as well as inflation forecasting. The main purpose of this exercise is to find consistent out-ofsample forecast based on mean square error minimization criteria, in which error volatility is minimized after training network with 10 hidden layers. Model simulation used feed forward with back propagation methodology which requires an activation function which used generalized delta rule. We have tried with various lags of 2, 4,6,8,10,12 and the mean square error was found to be least for lag 4. We have generated the step ahead forecast for our optimal case (lag 4).

ARIMA model

Finally, we have carried out inflation forecast for the months till March 2018. The best ARIMA forecasts (interval estimates, 95% confidence level), using WPI data till March 2017, is given below.

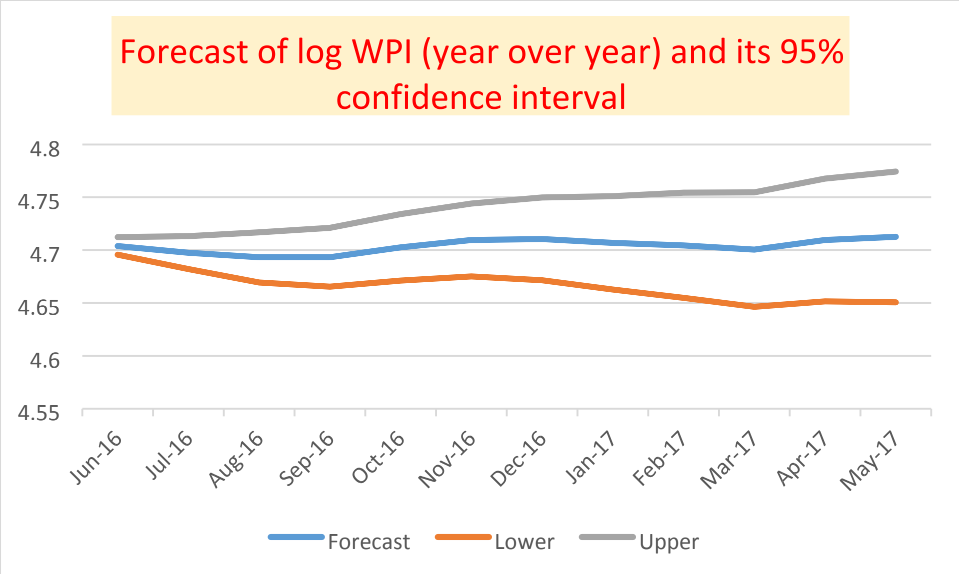

Fig1: Forecast of monthly year-over-year Inflation and its 95% confidence interval We have used a machine learning algorithm (artificial neural network) for the step ahead WPI and inflation forecasting. We have tried with various lags of 2, 4,6,8,10,12 and the mean square error was least in case of lag 4. We have generated the step ahead forecast for our optimal case (lag 4). The Fig1 and 2 below depicts the graphical representation.

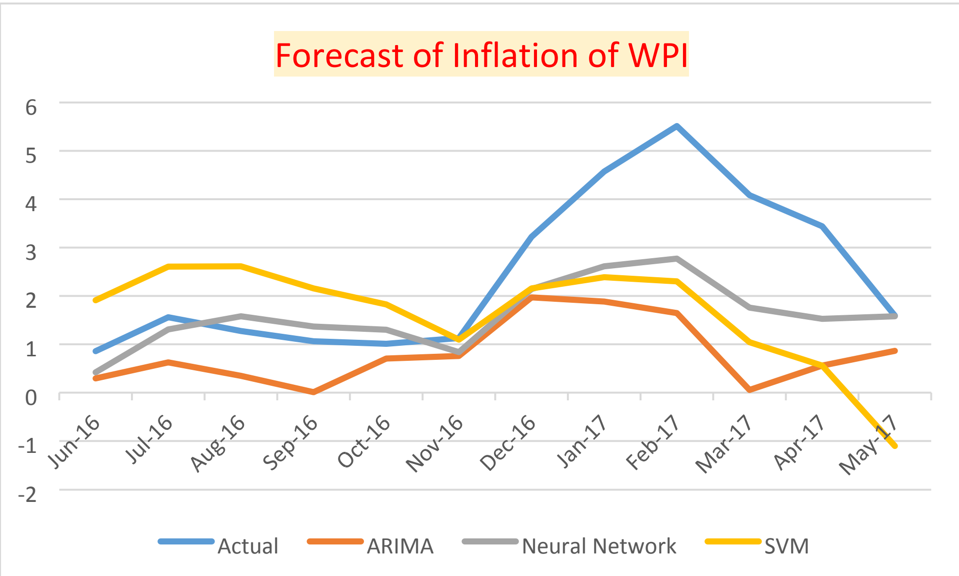

Fig2: Step ahead forecast of WPI core inflation using artificial neural network The below tables present the WPI and core inflation forecast using ARIMA and Artificial Neural Network.

Table1: Step ahead performance of WPI

Month	WPI(Actual)	ARIMA	ARIMA Lower	ARIMA Upper
Jul-17	4.70	4.70	4.69	4.71
Aug-17	4.70	4.69	4.68	4.71
Sep-17	4.70	4.69	4.66	4.71
Oct-17	4.70	4.69	4.66	4.72
Nov-17	4.70	4.70	4.67	4.73
Dec-17	4.71	4.70	4.67	4.74
Jan-18	4.72	4.71	4.67	4.74
Feb-18	4.73	4.70	4.66	4.75
Mar-18	4.74	4.70	4.02	4.65

Table2: Step ahead performance of WPI inflation

Month	WPI Inflation(Actual)	ARIMA	Neural Network	SVM
Jun-16	0.85	0.29	0.42	1.91
Jul-16	1.56	0.62	1.31	2.60
Aug-16	1.27	0.35	1.58	2.61
Sep-16	1.06	0.01	1.37	2.16
Oct-16	1.01	0.70	1.30	1.82

Nov-16	1.13	0.75	0.84	1.09
Dec-16	3.22	1.97	2.14	2.15
Jan-17	4.57	1.88	2.61	2.38
Feb-17	5.51	1.64	2.77	2.30
Mar-17	4.08	0.055	1.76	1.04
Apr-17	3.44	0.56	1.52	0.56
May-17	1.59	0.86	1.58	-1.09

The dataset has been divided in to training set with 90% of the time-series data, validation set with another 10% of the time-series data. The measurements available for the WPI parameter were collected between the years 2005 and 2017. The dataset consists of 156 averaged monthly observations. The system consists of composed of ARIMA, an intelligent hybrid model- ANN algorithm and also SVM. The results were presented in terms of measures of RMSE. Among the proposed models, ANN found to be the best configuration for estimation of WPI. The results depict the comparison of the forecast of three algorithms. From the graphs also it can be seen that ANN performs better compared to the ARIMA and SVM. Fig2 gives the forecast of WPI. The predicted results of three algorithms are presented in the table 2.

Conclusion

This paper used a univariate artificial neural network model to forecast monthly year over year inflation in India. The forecast was made for the period 2012-2017 using monthly series. The main purpose of the work was to generate forecasts that follow closely with actual data using three methods: ARIMA, ANN and SVM. The Nonlinear Autoregressive Network (NAR) model was trained with 20 hidden layer units, 1 output unit and backpropagation procedure. The forecast results remarkably indicate that ANN predict accurately. Finally, the comparison of the out-of-sample forecast performance of the ANNs with their econometric counterparts, SVM and ARIMA showed that the RMSE of the ANN was lower than those of ARIMA and SVM models. And so, judging by the RMSE criterion, forecast based on ANN are more accurate.

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REFERENCES

H. Marek., K. Michel, C. Josef, The application of structured feedforward neural networks to the modelling of daily series of currency in circulation, Working paper series 11, Czech National Bank, 2005.

T. Greg, H. Sarah, Forecasting GDP growth using Artificial Neural Networks, Working Paper 99-3, Bank of Canada, 1999

MOSPI, http://eaindustry.nic.in/home.asp

ARIMA model, https://people.duke.edu/~rnau/411arim.htm

Struvite Removal Using Ultasonic System

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ABSTRACT

Due to an increased need to treat wastewater from domestic and industrial sources to conform to Urban Waste Water Directive (91/271/EEC), wastewater treatment plants are faced with the deposition of Struvite causing pipe blockages and reduction in the rate of discharge flow thereby affecting cost and operation. The struvite is formed when the concentration of NH4+, Mg2+, and HPO43 ions reaches Supersaturation; however, its formation is controlled by pH, mixing energy, temperature and the presence of foreign ions in the solution. And when struvite is used as a fertilizer it displays good agricultural properties. Conventional methods for Struvite Removal like the use of highly corrosive chemicals, mechanically grinding pipe walls and hydro-jetting with high pressure streams of water may cause change in the quality of output water, pipe damages, leftover within the pipes and lengthy time and cost of operation. Following a review of current and previous literature, analysis was carried out to encourage the formation of a synthetic struvite and break it off using pressure waves from an ultrasonic system while at the same time finding out most appropriate pH for the formation of struvite and analyse the solubility of struvite and one of its derivatives. Results indicated that pressure waves can break off struvite coated on glass, plastic and metallic pipes within 2, 3 and 15 minutes respectively. While struvite is best formed at pH level of 9, Newberyite is less soluble than struvite due to loss of crystalline water and ammonia holding the Newberyite crystals. The use of ultrasonic system is therefore an effective and environmentally friendly method for the control of struvite from wastewater treatment plants. Meanwhile research is recommended to commercialise the method and also carry out environmental impacts assessment.

The Current Dynamics of Pakistan-US Relationship: Morphing from Being a Strategic Alliance to Transactional Relationship (An Overview)

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ABSTRACT

There is no permanent enemy and friend in an international politics, only national interest plays the pivotal role. The relationship between Pakistan-United States have been asymmetric and multifaceted. Since the creation of Pakistan in 1947, the relationship of Pakistan and United States has experienced many ebbs and flows. The relationship between these two unequal partners are based on divergent mutual perception.The main objectives of the research paper will be to analyse the factors of fluctuation of United States-Pakistan relations in different periods. In this research paper analytical and qualitative methodology will be applied to gauge the Pakistn-United States relations. The geo-political approach will be applied to assess transactional relations between two countries. The data collection will be based on primary and secondary sources i.e books, journals, interviews and newspapers etc. The reason behind this up and down relationship between these two nations are difference of attitude by the United States towards Pakistan at different times and occasions. As a matter of fact, the United States only helped Pakistan when its services were required for the fulfillment of its objectives in the region of South Asia and Southwest Asia. On the other hand, in this uneasy marriage which has further strained, there is no option of divorce for either nations despite growing estrangement. Breaking the relationship will go against both the nations. In order to extricate from Afghan quagmire, the United States require the vital support of Pakistan. Thus, despite the mistrust between these two countries, there is also a realisation that a complete break down of relationship between Islamabad and Washington will be in nobody's interest.

The present paper is an attempt to evaluate the factors responsible for bringing Pakistan and the US as a short term strategic allies at different occasions. Besides, the paper will also evaluate the main reason for shifting this strategic alliance between Pakistan and the United States into a transactional relationship. In the end, the vital strategies will be recommended for the improvement of relations between Pakistan and the United States.

HISTORICAL SUMMARY

Pakistan came into being as an independent state in 1947 in the violence of partition making India its most immediate and powerful threat. This insecurity drove Pakistan's search for allies and a position in world politics which were dominated by the emerging Cold War. These influences compelled Pakistan to abandon its original ideals of neutrality and Muslim solidarity, and to seek security through alignment with the US.

US interest in South Asia came from its strategy of containing the USSR to prevent communist expansion towards the Middle East and, later, to also contain China (Cheema, 1990:136). . After being rejected by India in 1949, which chose a non-aligned policy, it took another five years for the US to turn to Pakistan as its regional partner. Under the 1954 Mutual Defence Pact, Pakistan joined the US alliance system, receiving military aid in return for access to bases and military co-operation. However, the primary aims of the two states did not align. Whilst the US was arming Pakistan against the USSR, Pakistan's prime concern was India and a resolution of the Kashmir issue. These misaligned aims underlay the relationship from the start and contributed to its brittle and fluctuating quality.

Despite this, the alignment was stable up to the early 1960s. Pakistan joined other US-led regional alliances, SEATO and CENTO, and was recognised as a key US ally (Burke, 1973:171). It received large amounts of military aid in return and the US turned a blind eye when the military ousted the elected civilian governmentin October 1959. Differences surfaced, however, when the US armed India in its 1962 border dispute with China and they came to a head when Pakistan's military aid was withdrawn in the 1965 Indo-Pakistan War. Bitterly disappointed that the US had not supported it against India, Pakistan turned to China for alternative arms supplies (Reidal, 2010:14), but remained a member of the US-led alliances.

By 1970, Pakistan's connection with Beijing was used by Washington as part of its triangular diplomacy which involved improving relations with China and deliberately worrying the USSR in the process (Sharma, 1999:93). At the same time, East Pakistan was breaking away from West Pakistan resulting in brutal repression, to which the US again turned a blind eye. However, the US did not intervene to prevent East Pakistan's secession, which was won with Indian military help, though it did deter India from threatening West Pakistan. Pakistan again felt betrayed and diversified its foreign policy towards Islamic states and China, without again breaking off its relations with Washington.

India's 1974 nuclear test prompted Pakistan to accelerate its own nuclear programme, against the non-proliferation aims of the US. This created severe tension culminating in suspension of aid and Pakistan's withdrawal from CENTO (Arif, 1984: 346). In contrast to America's response to the earlier military take-over, General Zia's coup in July 1977 and the subsequent repression drew severe human rights criticism from the Carter administration. With US-Pakistan relations at a low ebb, the USSR invaded Afghanistan in 1979 and these concerns were put to one side asthe US attempted to regain Pakistan as a close ally. Initially refusing Carter's offer, Zia used his country's geo-strategic value to negotiate a better aid package from the Reagan administration and then helped to arm the Mujahidin in a proxy war against the USSR (Malik, 2001:361). In this, the US aim was to weaken and expel the USSR from Afghanistan. Pakistan shared this latter aim, but also wanted a sympathetic successor regimein Kabul which would allow military strategic depth against India and not incite nationalist sentiment in the border regions. The US was not particularly interested in these concerns of Pakistan.

Pakistan's nuclear programme continued throughout this period without noticeably affecting relations with Washington. However, when the USSR withdrew from Afghanistan in 1989 this concern over nuclear proliferation led to the US suspending aid once more, despite Pakistan's reversion to democratic government. Islamic terrorism was another issue between the two countries when the US accused Pakistan of sponsoring terrorists in Kashmir and Bosnia. In the Afghan civil wars, which followed the Soviet collapse,Islamabad sponsored the pro-Pakistan Taliban (Bassiouni, 2008:40). Eager for access to new gas fields in Central Asia, and to contain Iranian influence, the US initially joined Saudi Arabia in financing Taliban support. However, deteriorating human rights, drug trafficking and support for al-Qaeda made the US distance itself from the Taliban and criticise Pakistan over its sponsorship of them. In 1998, Pakistan responded to India's nuclear tests with tests of its own, resulting in another US aid embargo (Fry, 2013:136). Musharraf's 1999 military coup drew additional sanctions.

It was in this period that the 9/11 incident occurred. Embargos and sanctions were once again lifted and the US gave Pakistan a $2.64bn aid package in return for joining Washington in clearing al-Qaeda and the Taliban from Afghanistan. In 2004, Pakistan was declared a major non-NATO ally of the US but, here also, aims were not aligned. The US wanted to destroy al-Qaeda and their Taliban hosts but still did not share Islamabad's hope for a pro-Pakistan regime in their place. Pakistan was pressured by the US to abandon the Taliban, and India-friendly factions took Kabul and became prominent in government.

To escape the US military, the Taliban and other militant groups moved to the Pakistan border areas from where they attacked NATO in Afghanistan and created potential for instability in Pakistan itself. Under US pressure, the Pakistan military attacked those groups and this caused resentment and violence inside the country. Frustrated with the lack of progress, and suspicious of Pakistani collusion, the US also attacked those areas with drones, adding anti-American feeling to the existing resentment of the Pakistan military. Trust had broken down to such an extent that when the US found Osama binLaden on Pakistani territory in 2011 they mounted a military operation to kill him without even consulting Islamabad. Later the same year, NATO destroyed a military base at Salala killing 24 Pakistani soldiers (BBC News, 2011). In retaliation Pakistan blocked NATO access to Afghanistan which was not reopened until 2012 when Hillary Clinton issued her rather half-hearted apology which was referred to earlier. Despite all these problems, relations never broke down completely and the US continued to give aid to the Pakistan military.

CURRENT PAKISTAN-US RELATIONSHIP:

Recently the former Foreign Minister of Pakistan, Khawaja Asif said there is a "huge trust deficit" between Pakistan and the US (Kermani, 2017). This statement suggests that little has changed in US-Pakistan relations and the pattern of relations has become well set over sixty-five years and shows little sign of changing. Based on fundamental interests which diverge and sometimes conflict, it has been formed and re-formed through expedient opportunism and wilful blindness with an accumulated deficit of trust and mutual antipathy. It is resentfully held together by mutual reliance for non-mutual ends.

In 1962 Pakistan felt let down by the US support for India in its border war with China and the US felt let down by Pakistan's subsequent turn to China and development of nuclear weapons. The trust deficit grew in a cycle of on-off relations dominated by the Soviet invasion of Afghanistan, embargos and sanctions over nuclear weapons and human rights, the War on Terror and the US occupation of Afghanistan. It now focusses on US allegations of Pakistan's duplicity in protecting Afghan terrorists whilst also claiming to be Washington's allies and receiving its support.

As in the past, a new US administration has stirred up contradictions. When Trump was elected last year he inherited a legacy of disputes not just about harbouring terrorists but also about US demands for the release of Shakil Afridi (the doctor who helped lead the CIA to Osama bin Laden); (Gul, 2012:26) the withholding of $300m reimbursements to the Pakistani army; and barriers to letting Islamabad buy F16 fighter jets. In addition there were worries that Trump's business interests might lead him to favour India. Nevertheless, just after his election Trump spoke to then PM Nawaz Sharif and gave cause to believe he might bring a more constructive approach. He was reported as telling the PM, "You are doing amazing work which is visible in every way."and, "Your country is amazing with tremendous opportunities. Pakistanis are one of the most intelligent people." In what must have been interpreted as an offer to help over escalating tensions with India in Kashmir at that time he was reported to have added, "I am ready and willing to play any role that you want me to play to address and find solutions to the outstanding problems." (*The Guardian*, 1st December, 2016). However, eight months later there was a now familiar turn-around in rhetoric when Trump told his nation about his new strategy for Afghanistan (which was a reversal of his predecessor's and reverted to increasing US engagement with an unspecified number of soldiers, an unspecified timetable and an unspecified objective). One part of the strategy which was not new was blame for, and threats to, Pakistan: "We can no longer be silent about Pakistan's safe havens for terrorist organisations, the Taliban and other groups that pose a threat to the region and beyond."While acknowledging America's strong relations with Pakistan and its sacrifices to terrorism, Trump said Pakistan would be a "pillar" of his strategy and itwould have "much to lose" if it did not comply. (The *Guardian*, 22nd August, 2017). Aggravating an other sore for Pakistan he said he would encourage India to play more of a role.

In relation to this there have been some familiar responses from Pakistan. A senior Pakistani intelligence official was reported saying: "Pakistan itself is the victim of terrorism. We are fighting militants and have conducted many ground and aerial operations and destroyed their sanctuaries. We want to eradicate them physically and ideologically." (*The Guardian*, 22nd August, 2017) Going a step further Naeem Khalid Lodhi, a defence analyst and retired general, said the US was to blame for its own failures in Afghanistan and, "They are shifting blame to Pakistan. Pakistan should not remain silent against such US behaviour and we should work to build a new political and strategical bloc with big powers like Russia and China." In a more measured response Khwaja Asif insisted there are no "safe havens" in Pakistan and pointed out that, "They do not need our territory any more. Almost 40% of Afghan territory is now under the direct control of the Taliban" (Kermani, 2017) This claim has greater credibility now that the US itself estimates that Afghan government forces control less than 60% of territory (*The Guardian*, 22 Augus, 2017) and it demonstrates that conditions have changed and that Trump has less leverage to force any of the regional states to do his will. Khwaja Asif also pointed to another factor reducing US influence when he claimed that Pakistan only received a trickle of economic assistance from Washington, "We do not get any military hardware from them. We are not like in the past when we were their proxy." (Kermani, 2017). Linked to this Trump is also faced with growing Chinese influence in the region and in Pakistan in particular, notably through over $50 billion investment associated with CPEC (Kiani, 2016).

Trump's actions look muddled. There is nothing in his new Afghan "fight to win" strategy which has not been tried before and failed to decisively win. The record of the last 16 years shows that the Taliban and other anti-US groups can survive military force and that, in any case, Washington cannot be relied upon to maintain a consistent long-term strategy which will outlast the Taliban's. There is a possibility that Trump may be aiming to make some short-term impact to be able to negotiate with the Taliban from a position of greater strength. This would be consistent with his "deal-maker" approach and he hinted at it in his speech, "Someday, after an effective military effort, perhaps it will be possible to have a political settlement that includes elements of the Taliban in Afghanistan" (*Dawn*, 22 August, 2017). However, the resources he appears to be committing do not make this strategy a "surge", like the one in Iraq, which will seriously deplete the Taliban and push them into such a position and, in any case, the Taliban have shown no indication that they would negotiate from a position of weakness themselves. The situation most likely to motivate the parties to a negotiated settlement is an acceptance that a stalemate has been reached from which neither can win. Thus, without any clear military objective, Trump's approach risks escalation and mission-creep which such open-ended strategies are prone to.

Blaming Pakistan is nothing new but the circumstances in which it is being done have changed. Pakistan is less reliant on the US but the US is still reliant on Pakistan for land and air access to Afghanistan, for co-operation in the border regions and for intelligence sharing. Notably, though, the US cannot afford to let Pakistan become unstable because of the risks over who might get access to its nuclear arsenal. Against this background, therefore, it is unclear exactly what "getting tough" with Pakistan could entail. Military aid is limited and economic investment is dwarfed by China. Military action would alienate an already unsympathetic population and could alter the whole balance of power in the region further against the US. So, it is not clear how Trump would make good on his threat without inflicting harm on his own national interests.

Another change has been the increasing involvement in Afghanistan of other regional powers to fill the void left by the US withdrawal under Obama. This has allowed the Taliban to diversify its sources of support, especially since the emergence of Islamic State. In addition to Pakistan there is evidence that Russia, Iran, China and Saudi Arabia are all seeking influence with the Taliban. There is also evidence that Russia is allegedly supplying weapons and cash not just to the Taliban but also to local strongmen in northern Afghanistan causing further destabilisation (*The Guardian*, 22 October, 2017) Just as Pakistan protects its interests against India in Afghanistan so too these other states seek to protect their interests there against unacceptable Islamic groups or other rival states. So, picking out Pakistan as the main culprit seems both unfair and counter-productive, particularly as the proportion of Afghan terrorists sheltering in Pakistan is small, as you have identified.

However, Washington's continued blaming of Pakistan serves an important purpose for them: it creates a narrative acceptable to the domestic audience that the US military is not responsible for its failures. In this sense it protects the military and successive administrations from criticism. But this is likely to be counter-productive to wider US interests particularly in relation to China in that it will bring Beijing and Islamabad even closer together as allies.

Trump potentially alienated Pakistan further with his claim that he would encourage India to play more of a role in Afghanistan. This could also be counter-productive since it is the fear of a greater Indian presence there that motivates Pakistan's backing for Afghan militants, as a buffer against their regional enemy's influence. So, greater Indian influence will encourage greater Pakistani support for these groups. Not only will this alarm Islamabad but it will also concern China as a rival of India and with large investments and strategic interests in the region. This will further strengthen the convergence of interests between Pakistan and China and highlight the divergence of interests between Pakistan and the US.

Having created that situation it is by no means clear that India will back up Trump in the way he wants. Ajai Shukla, a writer on Indian strategic affairs claims, "Trump is ahead of Indian policy on this….with the situation in Doklam, and Kashmir on fire again….India is in no position to respond with sizable troop numbers in Afghanistan" (*The Guardian*, 22 August, 2017) Even if Trump is only looking for increased economic assistance it may not be forthcoming according to NandanUnnikrishnan, vice-president of the Observer Research Foundation, "I don't know how India is going to do that….. India has its own economic challenges for a variety of reasons, including demonetisation. And our economic growth is also forecast to slow down this year" (*The Guardian*, 22 August, 2017) India has a long history of knowing that the US desperately wants to be its premier regional ally yet holding Washington at arm's length and maintaining its independence of strategy and action. Modi is an Indian nationalist and will not allow his country to be manipulated by the US, and if Trump thinks he can solve his regional problems by mediating a solution in Kashmir he can forget it. India has not and will not accept external interference in this dispute, as Hilary Clinton and President Obama found out.

Thus, Trump appears to have stirred up regional fears and escalated tensions without creating the prospect of furthering US interests in any clear way. Pakistan, however, now has genuine options for hedging against the US by increasing its partnership with the rising World power that is China. As General Maj Gen Asif Ghafoor(Inter-Services Public Relations Director) told reporters, referring to Trump's decision, "Let it come… Even if it comes... Pakistan shall do whatever is best in the national interest " (*Dawn*, 22 August, 2017). As with all Trump policies, however, it remains to be seen whether he will stick to it or change his mind again. After all it was only a little over a year ago that he was arguing for a withdrawal from Afghanistan and telling Nawaz Sharif what amazing work Pakistan was doing.

On 1st January 2018, the US President Trump accused Pakistan of deceiving the United States while receiving billion of dollars in aid. This statement invited a series of response from Pakistan which further deteriorated the relationship. In his tweet he stated that the US has foolishly given more than $33 billion in aid for the last 15 years but Pakistan in response gives only lies and deceit to the US. He further accused Pakistan of giving safe haven to the terrorists the US hunt in Afghanistan (Aziz, 2018). On the same day of Trump's tweet, the Foreign Minister of Pakistan, Kawaja Asif in an interview to a private Pakistani channel said that Pakistan has already said no more to the US and therefore, Trumps no more is of no importance to Pakistan. Again on 3rd January 2018, Kawaja Asif, in response to the US President Trump's aggressive speeches against Pakistan, reminded the US administration through a series of tweets the services which Pakistan had rendered to the US particularly during the war against terror. In those tweets he addressed the US that history taught Pakistan not to blindly trust the US. A dictator (Musharraf) surrendered to the US in a single phone call. Pakistan witnessed a worst bloodshed, the US carried 57,800 attacks on Afghanistan from Pakistani bases. He further said that from soil of Pakistan, the US forces were supplied the arms and explosives. Thousands of Pakistani civilians and military soldiers became the victim of war which was initiated by the US (Iqbal, 2018). Pakistani leaders also responded by saying that that Pakistan has been made a scapegoat for their failure in Afghanistan.

CONCLUSION

The balance of gains from the relationship appears to strongly favour the US but there is little indication that Pakistan will give up on it in the foreseeable future. Despite American frustrations, the US would be unwise to abandon Pakistan as it did in the past since many of the circumstances favouring reverse influence will remain even after a partial withdrawal from Afghanistan.

The relationship between Pakistan and the US is not a strategic relationship but it was primarily a transactional relationship. There should be a rational, open approach in which both the nations must know what the expectations are. They must also seek to identify the areas where they can work together and try to isolate the areas where they cannot work together and promote those where understanding can be reached. Further, it is better for Pakistan to stop competing with the US-India relationship which, difficult though it might be to swallow, had actually now acquired a strategic dimension which is likely to be further intensified. Therefore, Pakistan should recognise the realities of diverging interests, accept the realities of US regional strategy and advocate a negotiated approach to transactional co-operation within these limits. It is an approach which has not often been present in the relationship and would require a considerable re-orientation of attitudes on both sides.

On the US side there have been two tensions in policy towards Pakistan which have bred instability in the relationship in addition to that caused by the underlying divergence of interests. First was the tension between wanting India as first choice for regional partner but needing Pakistan because of regional geo-politics. This led to a double game in which the US kept its India options open whilst engaging Pakistan and ultimately facilitated greater Indian influence in Afghanistan. In the Cold War it appeared Democrats favoured India while Republicans favoured Pakistan, but as India grew in power and significance this became a general preference for India. However, Washington's continued reliance on Pakistan will impede relations with India and its continued espousal of India will reinforce distrust in Pakistan. The second tension was between the promotion of liberal values and nuclear non-proliferation on the one hand, and support for illiberal regimes and tolerance of nuclear proliferation in pursuit of realist power politics on the other. In the case of Pakistan this led to sharp oscillations of policy and an expectation of mistrust. Unless the US can find a way of at least smoothing the change from one policy phase to the next this will remain a problem for the relationship. However, the legacy of mistrust and of popular anti-US and anti-Pakistan sentiment in each nation makes this even more difficult.

It would be rewarding to conclude that the history of US-Pakistan relations contains optimistic indications of how they might be put on a more constructive level. However, the pattern of relations has become well set over 70 years and shows little sign of changing. Based on fundamental interests which diverge and sometimes conflict, it has been formed and re-formed through an accumulated deficit of trust and mutual antipathy. It is resentfully held together by mutual reliance for non-mutual ends. The military-dominated Pakistani elite relies on US money and arms to confront India and to maintain state integrity. The US relies on them for access to and use of their geo-strategic location and intelligence and for keeping their nuclear arsenal safe from American enemies. Thus, despite a massive power disparity between the two states, Pakistan has been able to exert considerable reverse influence on the US to keep the money and arms coming. To change this pattern would require considerable institutional and popular changes in attitudes which are well engrained. Obama's failed attempt to turn a new page in 2008 illustrates how difficult such change was to make. This suggests more of the same, unless and until an external shock shakes regional relationships into a new configuration.

BIBLIOGRAPHY

Arif, K. (ed.) (1984), *America-Pakistan relations*, Vol. 1, Lahore: Vanguard Books Ltd.

Aziz, Saba (2018), "Pakistan-US war of words over Donald Trump's tweet", Aljazeera News, 2nd January, 2018 at https://www.aljazeera.com/news/2018/01/pakistan-war-words-donald-trump-tweet-180102055709366.htm

Bassiouni, M. C. (2008), *International Criminal Law: Sources, Subjects and Contents*, The Netherlands: Martinus Nijhoff Publishers, 2008.

BBC News, "Pakistan outrage after 'NATO attack kills soldiers'", 26th November 2011, at http://www.bbc.co.uk/news/world-asia-15901363

Burke, S. M. (1973), *Pakistan's Foreign Policy: A Historical Analysis*, London: Oxford University Press.

Cheema, P. I (1990), *Pakistan's Defence Policy, 1947-1958*, London: Macmillan Press.

Dawn, "Trump backs off Afghan withdrawal, lambastes Pakistan over terrorist 'safe havens'", 22nd August, 2017 at https://www.dawn.com/news/1353164

Fry, J.D. (2013), *Legal Resolution of Nuclear Non Proliferation Disputes*, Cambridge, University of Cambridge.

Gul, I. (2012), *Pakistan before and after Osama*, New Delhi: Roli Books Pvt. Ltd.

Iqbal, Anwar. (2018), "Trump's tweet on Pakistan sparks war of words", *Dawn*, 2nd January, 2018 at https://www.dawn.com/news/1380174

Kiani, Khaleeq (2016), "With a new Chinese loan, CPEC is now worth $51.5bn",*Dawn*, 30th September, 2016 at https://www.dawn.com/news/1287040

Kermani, Semundar, (2017), "Pakistan bemoans US 'trust deficit' over Afghan conflict", *BBC News*, 24th October, 2017 at http://www.bbc.co.uk/news/world-asia-41743165

Malik, I.H. (2001), "Pakistan in 2000: Starting Anew Stalemate?" *Asian Survey*, Vol. 41, No. 1, 2001.

Riedel, B., (2010), *The Search for Al Qaeda: Its Leadership, Ideology, and Future*, Washington: Brookings Institution, 2010.

Sharma, S. R., (1990), *India-China Relations: Friendship Goes with Power*, New Delhi: Discovery Publishing House.

The Guardian, "Russia accused of supplying Taliban as power shifts create strange bedfellows", 22nd October, 2017 at https://www.theguardian.com/world/2017/oct/22/russia-supplying-taliban-afghanistan

The Guardian, "'Terrific guy, fantastic country': Trump heaps praise on Pakistan's leader", 1st December, 2016 at https://www.theguardian.com/world/2016/dec/01/terrific-guy-fantastic-country-trump-pakistan-nawaz-sharif

The Guardian, "Trump's Afghan shift praised in Kabul but leaves Pakistan wary", 22nd August, 2017 at https://www.theguardian.com/world/2017/aug/22/donald-trump-kabul-praises-fight-to-win-afghanistan-strategy

The Guardian, "Why Trump's Afghanistan strategy risks the worst of both worlds", 22nd August, 2017 at https://www.theguardian.com/us-news/2017/aug/22/why-trumps-afghan-strategy-risks-the-worst-of-both-worlds

Service-Value-Loyalty Chain: A Theoretical Underpinning for Emerging Markets

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Abstract

The saturation of developed economies has resulted into shifting the focus of prominent marketing strategies towards the emerging markets. Past researchers reported that the gross domestic product (GDP) of these markets will surpass the aggregated GDP of all ageing and stagnant developed markets. This will enhance the attractiveness of the emerging markets. Further, the opening up of country's economies and economic liberalization amongst the developing markets will create growth opportunities for both the domestic and international marketers. According to researchers the emerging markets can be characterized by heterogeneity, role of socio-political governance, unbranded competition, shortage of resources related to production, exchange and consumption and infrastructural constraints. This contextual difference underlined by novel characteristics introduces new and specific research questions related to contextual shift and localized nature of emerging markets. These questions are need to explored and examined for better understanding of the context in order to check relevance of contemporary marketing practices. Taking the aforementioned aspects into consideration the marketing researchers need to develop new ideas and concepts and re-test the established ideas. The marketing researchers may leveraged this opportunity by introducing rethinking marketing strategies for checking and enhancing appropriateness with the emerging market context. Along with the contextual shifts, the customer-centric aspects like customer value, customer satisfaction and customer loyalty could be research avenues with special reference to emerging markets. Taking heterogeneity of the emerging markets into consideration it becomes important to analyze the

impact of perceived value on customer loyalty. In case of services the well-researched variables like reliability, responsiveness, assurance, empathy and tangibles could be important constructs that leads to perceived value. Further, according to a report published by the Internet and Mobile Association of India (IAMAI) the number of internet users is going to grow by 11.34 percent and will reach up to 500 million. This intern will increase the mobile application (App) users by few percent. According to STATISTA the statistical portal in India the taxi aggregators like Ola and Uber have appointed about 9 lakh drivers responding to the enhanced demand. Even though the service providers are addressing the marketing needs the customers were found unhappy as per an online survey. These aforementioned facts indicate the scope and need for researching service-value-loyalty chain in the emerging markets with special reference to mobile application based services. Therefore, this study did the theoretical underpinning of state of art literature relevant to service-value-loyalty chain. As a knowledge contribution this study proposed a conceptual model to examine impact of e-service quality perception on perceived value. The research context is limited to the mobile application users.

Asymptotic Theory of Bayes Factor in Stochastic Differential Equations With Respect to Time and Individuals

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Abstract

The problem of model selection in the context of a system of stochastic differential equations (*SDE*'s) has not been touched upon in the literature. Indeed, properties of Bayes factors have not been studied even in single *SDE* based model comparison problems.

In this article, we first develop an asymptotic theory of Bayes factors when two *SDE*'s are compared, assuming the time domain expands. Using this we then develop an asymptotic theory of Bayes factors when systems of *SDE*'s are compared, assuming that the number of equations in each system, as well as the time domain, increase indefinitely. Our asymptotic theory covers situations when the observed processes associated with the *SDE*'s are independently and identically distributed (*iid*), as well as when they are independently but not identically distributed (non-*iid*). Quite importantly, we allow inclusion of available time-dependent covariate information into each *SDE* through a multiplicative factor of the drift function in a random effects set-up; different initial values for the *SDE*'s are also permitted.

Thus, our general model-selection framework includes simultaneously the variable selection problem associated with time-varying covariates, as well as choice of the part of the drift function free of covariates. It is to be noted that given that the underlying process is wholly observed, the diffusion coefficient becomes known, and hence is not involved in the model selection problem.

For both *iid* and non-*iid* set-ups we establish almost sure exponential convergence of the Bayes factor. As we show, the Bayes factor is inconsistent for comparing individual

SDE's, in the sense that the log-Bayes factor converges only in expectation, while the relevant variance does not converge to zero. Nevertheless, it has been possible to exploit this result to establish almost sure exponential convergence of the Bayes factor when, in addition, the number of individuals are also allowed to increase indefinitely.

We carry out simulated and real data analyses to demonstrate that Bayes factor is a suitable candidate for covariate selection in our *SDE* models even in non-asymptotic situations.

Keywords: Bayes factor consistency; Kullback-Leibler divergence; Martingale; Stochastic differential equations; Time-dependent covariates and random effects; Variable selection.

Introduction

Stochastic differential equations (*SDE*'s) have important standing in statistical applications where "within" subject variability is caused by some random component varying continuously in time. It also seems worthwhile to incorporate available time-dependent covariate information into the subject-wise *SDE*'s. Apart from the covariates there may also be random effects associated with the individuals, which may be useful in modeling variabilities between the individuals.

SDE-based models with time-dependent covariates are considered in Oravecz et al. (2011), Overgaard et al. (2005), Leander et al. (2015); moreover, Oravecz et al. (2011) analyse their covariate-based SDE model in the hierarchical Bayesian paradigm. In the literature, random effects SDE models without covariates seem to be more popular than those based on covariates. A brief overview of random effects SDE models is provided in Delattre et al. (2013) who undertake theoretical and classical asymptotic investigation of a class of random effects models based on SDE's. Specifically, they model the i-th individual by

$$dX_i(t) = b(X_i(t),\varphi_i)dt + \sigma(X_i(t))dW_i(t), \qquad (1.1)$$

where, for $i = 1,...,n$, $X_i(0) = x^i$ is the initial value of the stochastic process $X_i(t)$, which is assumed to be continuously observed on the time interval $[0,T_i]$; $T_i > 0$ assumed to be known. The function $b(x,\phi)$, which is the drift function, is a known, real-valued function on $R \times R^d$ (R is the real line and d is the dimension), and the function $\sigma : R \to R$ is the known diffusion coefficient. The *SDE*'s given by (1.1) are driven by independent standard Wiener processes $\{W_i(\cdot);\ i = 1,...,n\}$, and $\{\varphi_i;\ i = 1,...,n\}$, which are to be interpreted as the random effect parameters associated with the *n* individuals, which are assumed by Delattre *et al.* (2013) to be independent of the Brownian motions and independently and identically distributed (*iid*) random variables with some common distribution. For the sake of convenience Delattre *et al.* (2013) (see also Maitra and Bhattacharya (2016) and Maitra and Bhattacharya (2015)) assume $b(x,\varphi_i) = \varphi_i b(x)$. Thus, the random effect is a multiplicative factor of the drift function. In this work, we generalize this to a random effects *SDE* set-up consisting of time-dependent covariates.

Note that model selection constitutes an important part of research in both Bayesian and classical paradigms; see, for example, Dey *et al.* (2000), Jiang (2007), Claeskens and Hjort (2008), Muller¨ *et al.* (2013). In the case of *SDE*-based mixed effects models as well, model selection constitutes an important issue involving the choice of the drift function and selection of the appropriate subset of (time dependent) covariates. Here Bayes factors are expected to play the central role as their effectiveness in model selection in complex problems is well-established (see, for example, Kass and Raftery (1995) for a good account of Bayes factors). Unavailability of closed form expressions in the traditional *SDE* set-ups usually prompt usage of numerical approximations based on Markov chain Monte Carlo or related criteria such as the Akaike Information Criterion (Akaike (1973)) and Bayes Information Criterion (Schwarz (1978)). But we are not aware of any research existing in the literature that attempts to address covariate selection in *SDE*'s.

We are also not aware of any existing literature on asymptotic investigation of Bayes factors in the *SDE* context although Sivaganesan and Lingham (2002) present some asymptotic investigation of intrinsic and fractional Bayes factors in the context of three specific diffusion models. The only investigation available in this context seems to be that of Maitra and Bhattacharya (2018) which establish almost sure exponential convergence of Bayes factor in both *iid* and non-*iid* situations when the number of individuals increase without bound. Note that, the iid set-up is the case when there is no covariate associated with the model and when the initial values and the domains of observations are the same for every individual. The non-iid set-up, on the other hand, consists of timevarying covariates, different initial values and domains of observations;

In this article, we prove almost sure exponential convergence of the relevant Bayes factors in both *iid* and non-*iid* cases, assuming that the number of individuals, as well as the domains of observations, increase without bound. Incorporation of random effects is asymptotically feasible in our current asymptotic framework along with different sets of time-dependent covariates for different individuals.

It is important to remark that the diffusion coefficient becomes known once the continuous process is completely observed; see Roberts and Stramer (2001). Hence we assume that the diffusion coefficient is known, and is not involved in the model selection problem.

We begin by establishing an asymptotic theory of Bayes factor for two competing individual *SDE*'s, and then extend the theory to systems of *SDE*'s. In this context it is important to draw attention to the fact that even this relatively simple problem of comparing any two individual *SDE*'s using Bayes factors has not yet been considered in the literature. Our investigation in this simpler case, however, faced with an apparently negative result; the associated Bayes factor failed to be consistent in the sense that the relevant variance failed to converge to zero, even though convergence of the log-Bayes factor in expectation is ensured. Despite this, we have been able to utilise this result to establish almost sure exponential convergence of the Bayes factor when the number of individuals are also allowed to increase indefinitely.

The rest of our article is structured as follows. We begin with formalization of our set-up in Section 2. In Section 3 we state the asymptotics of Bayes factor for comparing two individual *SDE*'s. In Section 4 we exploit the asymptotic theory of Bayes factors developed for comparing individual *SDE*'s to construct a convergence theory of Bayes factors comparing systems of *SDE*'s in both *iid* and non-*iid* cases. Section 5 contains the details of two simulation studies and a real data analysis of company-wise national stock exchange data set to show the validity and usefulness of the Bayes factor in practical *SDE*-based models. We summarize our contributions and provide concluding remarks in Section 6.

For the assumptions, lemmas, verifications of theorem with proof and all technical details we refer to **arXiv:1504.00002v5** .

Formalization of the model selection problem in the *SDE* set-up when $n \to \infty$ and $T_i \to \infty$ for every i

That the systems considered by us are well-defined and the exact likelihoods are computable, are guaranteed by assumption (H2'') in Section 3. For our purpose we consider the filtration $(\mathcal{F}_t^W, t \geq 0)$, where $\mathcal{F}_t^W = \sigma(W_i(s), s \leq t)$. Each process W_i is a $(\mathcal{F}_t^W, t \geq 0)$-adapted Brownian motion.

Here we consider the set-up where, for $i = 1,2,\ldots,n$,

$$dX_i(t) = \phi_{i,\boldsymbol{\xi}_0^{(i)}}(t) b_{\boldsymbol{\beta}_0^{(i)}}(t, X_i(t))dt + \sigma(t, X_i(t))dW_i(t) \quad (2.1)$$

and

$$dX_i(t) = \phi_{i,\boldsymbol{\xi}_1^{(i)}}(t) b_{\boldsymbol{\beta}_1^{(i)}}(t, X_i(t))dt + \sigma(t, X_i(t))dW_i(t), \quad (2.2)$$

where, $X_i(0) = x^i$ is the initial value of the stochastic process $X_i(t)$, which is assumed to be continuously observed on the time interval $[0,T_i]$; $T_i > 0$. We consider (2.1) as representing the true model and (2.2) is any other model.

Inclusion of time-dependent covariates

We model $\phi_{i,\xi_j}(t)$ for $j = 0,1$, and $i = 1,\ldots,n$,as

$$\phi_{i,\boldsymbol{\xi}_j^{(i)}}(t) = \phi_{i,\boldsymbol{\xi}_j^{(i)}}(\boldsymbol{z}_i(t)) = \xi_{0j}^{(i)} + \xi_{1j}^{(i)} g_1(z_{i1}(t)) + \xi_{2j}^{(i)} g_2(z_{i2}(t)) + \cdots + \xi_{pj}^{(i)} g_p(z_{ip}(t)) \quad (2.3)$$

where $z_i(t) = (z_{i1}(t), z_{i2}(t), \ldots, z_{ip}(t))$ is the set of available covariate information corresponding to the i-th individual, depending upon time t. We assume $z_i(t)$ is continuous in t, $z_{il}(t) \in Z_l$ where Z_l is compact and $g_l : Z_l \to \mathbb{R}$ is continuous, for $l = 1,\ldots,p$. We let $Z = Z_1 \times \cdots \times Z_p$, and $\mathcal{Z} = \{z(t) \in Z : t \in [0,\infty)$ such that $z(t)$ is continuous in $t\}$. Hence, $z_i \in \mathcal{Z}$ for all i.

The random effects set-up

In (2.1) $\boldsymbol{\theta}_0^{(i)} = \left(\beta_0^{(i)}, \xi_{00}^{(i)}, \xi_{10}^{(i)}, \ldots, \xi_{p0}^{(i)}\right) = \left(\beta_0^{(i)}, \boldsymbol{\xi}_1^{(i)}\right)$stands for the true parameters, and $\boldsymbol{\theta}_1^{(i)} = \left(\boldsymbol{\beta}_1^{(i)}, \xi_{01}^{(i)}, \xi_{11}^{(i)}, \ldots, \xi_{p1}^{(i)}\right) = \left(\boldsymbol{\beta}_1^{(i)}, \boldsymbol{\xi}_1^{(i)}\right)$ are the parameters associated with (2.2). Let $\boldsymbol{\theta}_j^{(i)} \in \boldsymbol{\Theta} = \mathfrak{B} \times \boldsymbol{\Gamma}$ for all i, where both $\mathfrak{B}$ and $\boldsymbol{\Gamma}$ are compact spaces. We also assume that for $i = 1,2,\ldots,n$,

$\boldsymbol{\theta}_1^{(i)} \overset{iid}{\sim} \pi,$

where π is some specified distribution on $\boldsymbol{\Theta}$.

Hence, the above describes a random effects set-up. Observe that if $\xi_{lj}^{(i)} = 0$ for $l = 1, \ldots, p$, and for $i = 1,\ldots,n$, then it reduces to the random effects model of Delattre *et al.* (2013), showing that the latter is a special case of our model.

As is well-known, even though the term "prior" is not appropriate for the random effects coefficients, operationally there is no difference between a prior and a distribution for random effects in the Bayesian paradigm. Somewhat abusing the terminology, we continue to refer to the distribution of the *iid* random effects coefficients, π, as the relevant prior.

Covariate and drift function selection

Here, for every individual, there is an independent model selection problem. In other words, for each i, one needs to choose between $\boldsymbol{\theta}_0^{(i)}$ and $\boldsymbol{\theta}_1^{(i)}$. This involves selection of perhaps different sets of covariates for different i with respect to the coefficients $\boldsymbol{\xi}_j^{(i)}$, and different drift functions $b_{\boldsymbol{\beta}_j^{(i)}}$. Obviously, the dimensions of $\boldsymbol{\xi}_0^{(i)}$ and $\boldsymbol{\xi}_1^{(i)}$ are allowed to differ for each i; likewise, for every i, the dimensions of $\boldsymbol{\beta}_0^{(i)}$ and $\boldsymbol{\beta}_1^{(i)}$ may be different as well.

Form of the Bayes factor in our set-up

For $j = 0,1$, we first define the following quantities:

$$U_{i,\boldsymbol{\theta}_j^{(i)}} = \int_0^{T_i} \frac{\phi_{i,\boldsymbol{\xi}_j^{(i)}}(s) b_{\boldsymbol{\beta}_j^{(i)}}(s, X_i(s))}{\sigma^2(s, X_i(s))} dX_i(s), \qquad V_{i,\boldsymbol{\theta}_j^{(i)}} = \int_0^{T_i} \frac{\phi^2_{i,\boldsymbol{\xi}_j^{(i)}}(s) b^2_{\boldsymbol{\beta}_j^{(i)}}(s, X_i(s))}{\sigma^2(s, X_i(s))} ds \quad (2.4)$$

for $j = 0,1$ and $i = 1,\ldots,n$.

Let C_{T_i} denote the space of real continuous functions $(x(t), t \in [0,T_i])$ defined on $[0,T_i]$, endowed with the σ-field $\mathcal{C}_{T_i}$ associated with the topology of uniform convergence on $[0,T_i]$. We consider the distribution $P_j^{x_i,T_i,z_i}$ on $(C_{T_i}, \mathcal{C}_{T_i})$ of $(X_i(t), t \in [0,T_i])$ given by (2.1) and (2.2) for $j = 0,1$. We choose the dominating measure P_i as the distribution of (2.1) and (2.2) with null drift. So, for $j = 0,1$,

$$\frac{dP_j^{x_i,T_i,\boldsymbol{z}_i}}{dP_i} = f_{i,\boldsymbol{\theta}_j^{(i)}}(X_i) = \exp\left(U_{i,\boldsymbol{\theta}_j^{(i)}} - \frac{V_{i,\boldsymbol{\theta}_j^{(i)}}}{2}\right), \qquad (2.5)$$

where $f_{i,\boldsymbol{\theta}_0^{(i)}}$ denotes the true density and $f_{i,\boldsymbol{\theta}_1^{(i)}}$ stands for the other density associated with the modeled *SDE*. For each $i = 1,\ldots,n$, letting $X_{i,a,b}$ denote the *i*-th process observed on $[a,b]$ for any $0 \leq a < b < \infty$

$$I_{x^i,T_i,\boldsymbol{z}_i} = \int_{\boldsymbol{\Theta}} \frac{f_{i,\boldsymbol{\theta}_1^{(i)}}(X_{i,0,T_i})}{f_{i,\boldsymbol{\theta}_0^{(i)}}(X_{i,0,T_i})} \pi\left(\boldsymbol{\theta}_1^{(i)}\right) d\boldsymbol{\theta}_1^{(i)} \qquad (2.6)$$

denotes the Bayes factor associated with the *i*-th equation of the above two systems of equations. Assuming that the *SDE*'s (2.1) and (2.2) are independent for $i = 1,\ldots,n$,

$$I_{n,T_1,\ldots,T_n} = \prod_{i=1}^{n} I_{x^i,T_i,\boldsymbol{z}_i}$$

is the Bayes factor comparing the entire systems of *SDE*'s (2.1) and (2.2).

Comparisons between a collection of different models using Bayes factor, none of which may be the true model, is expected to favour that model which minimizes the Kullback-Leibler divergence from the true model.

The *iid* and the non-*iid* cases

We are interested in studying the properties of $I_{n,T_1,\ldots,T_n}$ in both *iid* and non-*iid* cases when $n \to \infty$ and $T_i \to \infty$. In the *iid* set-up, we assume that $x^i = x$, $T_i = T$ and $\boldsymbol{\theta}_j^{(i)} = \left(\boldsymbol{\beta}_j^{(i)}, \xi_{0j}^{(i)}\right)$, for $i = 1,\ldots,n$ and $j = 0,1$. In the non-*iid* case we relax these assumptions. However, for simplicity, we assume $T_i = T$ for each *i*, even in the non-*iid* set-up, so that in our asymptotic framework we study convergence of

$$\tilde{I}_{n,T} = \prod_{i=1}^{n} I_{i,T}$$

as $n \to \infty$ and $T \to \infty$, where $I_{i,T} = I_{x^i,T,z_i}$.

Main result on convergence of Bayes factor when two individual *SDE*'s are compared

From the system of *SDE*'s defined by (2.1) and (2.2) we now consider the *i*-th individual only. To avoid notational complexity we denote X_i simply by X. Consequently, $\varphi_{i,\xi j}(t)$ and T_i will be denoted by $\varphi_{\xi j}(t)$ and T, respectively. In connection with the *i*-th individual we consider the following two *SDE*'s:

$$dX(t) = \varphi_{\xi 0}(t)b_{\beta 0}(t,X(t))dt + \sigma(t,X(t))dW(t) \quad (3.1)$$

and

$$dX(t) = \varphi_{\xi 1}(t)b_{\beta 1}(t,X(t))dt + \sigma(t,X(t))dW(t). \quad (3.2)$$

For any $t \in [0,T]$, for $j = 0,1$, let

$$U_{\boldsymbol{\theta}_j,t} = \int_0^t \frac{\phi_{\boldsymbol{\xi}_j}(s)b_{\boldsymbol{\beta}_j}(s,X(s))}{\sigma^2(s,X(s))}dX(s), \quad V_{\boldsymbol{\theta}_j,t} = \int_0^t \frac{\phi^2_{\boldsymbol{\xi}_j}(s)b^2_{\boldsymbol{\beta}_j}(s,X(s))}{\sigma^2(s,X(s))}ds,$$
$$V_{\boldsymbol{\theta}_0,\boldsymbol{\theta}_j,t} = \int_0^t \frac{\phi_{\boldsymbol{\xi}_j}(s)b_{\boldsymbol{\beta}_j}(s,X(s))\phi_{\boldsymbol{\xi}_0}(s)b_{\boldsymbol{\beta}_0}(s,X(s))}{\sigma^2(s,X(s))}ds. \quad (3.3)$$

Note that $V_{\theta_0,t} = V_{\theta_0,\theta_0,t}$ and $V_{\theta_1,t} = V_{\theta_1,\theta_1,t}$. We also let

$$f_{\boldsymbol{\theta}_j,t}(X_{0,t}) = \exp\left(U_{\boldsymbol{\theta}_j,t} - \frac{V_{\boldsymbol{\theta}_j,t}}{2}\right). \quad (3.4)$$

Here we are interested in asymptotic properties of the Bayes factor, given by

$$I_T = \int \frac{f_{\boldsymbol{\theta}_1,T}(X_{0,T})}{f_{\boldsymbol{\theta}_0,T}(X_{0,T})}\pi(d\boldsymbol{\theta}_1), \quad (3.5)$$

as $T \to \infty$.

To compare between two individuals, we define $I_0 \equiv 1$ and for $t > 0$, let us define, analogous to (4.5),

$$I_t = \int \frac{f_{\boldsymbol{\theta}_1,t}(X_{0,t})}{f_{\boldsymbol{\theta}_0,t}(X_{0,t})}\pi(d\boldsymbol{\theta}_1). \quad (3.6)$$

Regarding convergence of I_T, we obtain following theorem under certain assumptions and lemmas.

Theorem 1 Assume the SDE set-up. Then under certain assumptions

$$\frac{1}{T}E_{\boldsymbol{\theta}_0}(\log I_T) \rightarrow -\delta, \quad (3.7)$$

but

$$\frac{1}{T^2}Var_{\boldsymbol{\theta}_0}(\log I_T) = O(1), \quad (3.8)$$

as T → ∞.

Asymptotic convergence of Bayes factor in the *SDE* set-up with respect to number of individuals and time

Convergence of Bayes factor in the *iid* set-up

Although Theorem 1 fails to ensure consistency of the Bayes factor as *T* → ∞ in the sense that the relevant variance is asymptotically positive, the theorem is useful to prove almost sure consistency when *T* → ∞ as well as *n* → ∞, for both *iid* and non-*iid* situations. Theorem 2 formalizes this for the *iid* set-up, while Theorem 3 establishes almost sure consistency of

the Bayes factor in the non-*iid* situation.

Theorem 2 Assume the iid set-up; also assume certain conditions hold for each SDE in the systems (2.1) and (2.2). Then

$$\frac{1}{nT}\log \tilde{I}_{n,T} \rightarrow -\delta, \quad (4.1)$$

almost surely, as n → ∞ and T → ∞.

Convergence of Bayes factor in the non-*iid* set-up

We now relax the assumptions $x^i = x$ and $\xi_{1j}^{(i)} = \xi_{2j}^{(i)} = \xi_{3j}^{(i)} = \cdots = \xi_{pj}^{(i)} = 0$ for $j = 0, 1$ Thus, we are now in a non-*iid* situation where the processes $X_i(\cdot)$; i = 1,...,n, are independently, but not identically distributed. As mentioned in Section 2.1 we assume that $\boldsymbol{\theta}_1^{(i)} \overset{iid}{\sim} \pi$. In this set-up, for each z ∈ Z = {z(*t*) ∈ Z : *t* ∈ [0,∞)}, it holds, due to Theorem 1, that

$$\frac{1}{T}E_{\boldsymbol{\theta}_0}\left(\log I_{x,T,\boldsymbol{z}}\right) \rightarrow -\delta(x,\boldsymbol{z}), \quad (4.2)$$

as $T \rightarrow \infty$, where $\delta(x,z)$ depends upon the initial value $x \in X$ and the set of time-dependent covariates $z \in Z$. The lemma in the supplement shows that $\delta(x,z)$ is continuous in $(x,z) \in X \times Z$. We define

$$\delta^{\infty} = \lim_{n\rightarrow\infty}\frac{1}{n}\sum_{i=1}^{n}\delta(x^i,\boldsymbol{z}_i).$$

(4.3)

Under the required assumptions and lemmas we have the following theorem in non-iid set-up.

Theorem 3 Assume the non-iid set-up, and all relevant conditions , for each SDE in the systems (2.1) and (2.2). Then

$$\frac{1}{nT}\log \tilde{I}_{n,T} \rightarrow -\delta^{\infty}, \quad (4.4)$$

almost surely, as $T \rightarrow \infty$ and $n \rightarrow \infty$.

Simulation studies

5.1 Covariate selection when n=1, T =5

We first demonstrate with simulation study the finite sample analogue of Bayes factor analysis associated with a single individual, when $T \rightarrow \infty$. In this regard, we consider modeling a single individual by

$$dX(t) = (\xi_1 + \xi_2 z_1(t) + \xi_3 z_2(t) + \xi_4 z_3(t))(\xi_5 + \xi_6 X(t))dt + \sigma dW(t), \quad (5.1)$$

where we fix our diffusion coefficient as σ = 20. We consider the initial value $X(0) = 0$ and the time interval $[0,T]$ with $T = 5$.

To achieve numerical stability of the marginal likelihood corresponding to data we choose the true values of ξ_i; $i = 1,\ldots,6$ as follows: $\xi_i \sim N(\mu_i, 0.001^2)$, where $\mu_i \sim N(0,1)$. Both ξ_i and μ_i have iid distribution.This is not to be interpreted as the prior; this is just a means to set the true values of the parameters of the data-generating model.

We assume that the time dependent covariates $z_i(t)$ satisfy the following *SDEs*

$$dz_1(t) = (\tilde{\theta}_1 + \tilde{\theta}_2 z_1(t))dt + dW_1(t)$$

$$dz_2(t) = \tilde{\theta}_3 dt + dW_2(t)$$

$$dz_3(t) = \tilde{\theta}_4 z_3(t)dt + dW_3(t) \quad (5.2)$$

where $W_i(\cdot)$; $i = 1,2,3$, are independent Wiener processes, and $\tilde{\theta}_i \stackrel{iid}{\sim} N(0,0.01^2)$ for $i = 1,\cdots,4$.

We obtain the covariates by first simulating $\tilde{\theta}_i \stackrel{iid}{\sim} N(0,0.01^2)$ for $i = 1,\cdots,4$, fixing the values, and then by simulating the covariates using the *SDE*s (5.2) by discretizing the time interval [0,5] into 500 equispaced time points. In all our applications we have standardized the covariates over time so that they have zero means and unit variances.

Once the covariates are thus obtained, we assume that the data are generated from the (true) model where all the covariates are present. For the true values of the parameters, we simulated $(\xi_1,\ldots,\xi_6)$ from the prior and treated the obtained values as the true set of parameters θ_0. We then generated the data using (5.1) by discretizing the time interval [0,5] into 500 equispaced time points.

As we have three covariates so we will have $2^3 = 8$ different models. Denoting a model by the presence and absence of the respective covariates, it then is the case that (1,1,1) is the true, data generating model, while (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), and (1,1,0) are the other 7 possible models.

As per our theory, for a single individual, the Bayes factor is not consistent for increasing time domain. However, we have shown that

$$\frac{1}{T}E_{\boldsymbol{\theta}_0}(\log I_T) \to -\delta$$

as $T \to \infty$. Thus, the Bayes factor is consistent with respect to the expectation. Our simulation results show that this holds even for the time domain [0,5], where we approximate the expectation with the average of 1000 realizations of I_T associated with as many simulated data sets.

5.1.1 Case 1: the true parameter set θ_0 is fixed

Prior on θ

We first obtain the maximum likelihood estimator (*MLE*) of θ using simulated annealing and then consider a normal prior with the *MLE* as the mean and variance $0.8^2 I_6$, where I_6 is the identity matrix of order 6.

Form of the Bayes factor

In this case the related Bayes factor has the form

$$I_T = \int \frac{f_{\boldsymbol{\theta}_1,T}(X_{0,T})}{f_{\boldsymbol{\theta}_0,T}(X_{0,T})} \pi(d\boldsymbol{\theta}_1), \quad (5.3)$$

where $\theta_0 = (\xi_{0,1}, \xi_{0,2}, \xi_{0,3}, \xi_{0,4}, \xi_{0,5}, \xi_{0,6})$ is the true parameter set and $\theta_1 = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)$ is the unknown set of parameters corresponding to any other model. Table 1 describes the results of our Bayes factor analyses. It is clear from the 7 values of the table that the correct model (1,1,1) is always preferred.

Table 1: Bayes factor results

Model	Averaged $\frac{1}{5}\log I_5$
(0,0,0)	-2.5756029
(0,0,1)	-0.913546
(0,1,1)	-0.5454860
(0,1,0)	-0.763952
(1,0,0)	-2.5774163
(1,0,1)	-0.9312218
(1,1,0)	-0.7628154

5.1.2 Case 2: the parameter set θ_0 is random and has the prior distribution π

As before, we consider the same form of the prior as in Section 5.1.1, but with variance $0.1^2 I_6$. In this case we calculate marginal likelihood of the 8 possible models, and approximate

$$\frac{1}{5} E_{\boldsymbol{\theta}_0}\left(\log \int f_{i,\boldsymbol{\theta}_1,5}(X_{0,5})\pi(d\boldsymbol{\theta}_1)\right)$$

for *i* = 1,...,8 by averaging over 1000 replications of the data obtained from the true model. Denoting its values by l_i, Table 2 shows that l_8 is the highest, implying consistency of the averaged Bayes factor.

Table 2: Averages of $\frac{1}{5}\times$ marginal log-likelihood

Model	`i
(0,0,0)	-1.21923
(0,0,1)	-0.21428
(0,1,0)	1.47992
(0,1,1)	2.102966
(1,0,0)	-1.222362
(1,0,1)	-0.21898
(1,1,0)	1.459921
(1,1,1)	2.121237 (true model)

5.2 Bayes factor analysis for *n*=15 and *T* =5

In this case we allow our parameter and the covariate sets to vary from individual to individual. We consider 15 individuals modeled by

$$dX_i(t) = (\xi_1^i + \xi_2^i z_1(t) + \xi_3^i z_2(t) + \xi_4^i z_3(t))(\xi_5^i + \xi_6^i X_i(t))dt + \sigma_i dW_i(t) \quad (5.4)$$

for $i = 1, \cdots, 15$. We fix our diffusion coefficients as $\sigma_{i+1} = \sigma_i + 5$ for $i = 1 \cdots, 14$ where $\sigma_1 = 10$. We consider the initial value $X(0) = 0$ and the interval $[0,T]$, with $T = 5$. As before, we generated the observed data after discretizing the time interval into 500 equispaced time points. Here our covariates and the parameter set $\boldsymbol{\theta}_0^i = (\xi_{0,1}^i, \xi_{0,2}^i, \xi_{0,3}^i, \xi_{0,4}^i, \xi_{0,5}^i, \xi_{0,6}^i); i = 1, \ldots, 15$, are simulated in a similar way as mentioned in Section 5.1.

For each of the 15 individuals, the true set of covariate combination is randomly selected. Thus, for a given model, there are 15 sets of covariate combinations to be compared with other models consisting of 15 different sets of covariate combinations. To decrease computational burden we compare the true model with 100 other models consisting of different sets of covariate combinations.

The Bayes factor corresponding to the *j*-th covariate combination is given by

$$I_{nT}^{j} = \prod_{i=1}^{n} \int \frac{f_{i,\boldsymbol{\theta}_1^{(i)}}^{j}(X_{i,0,T})}{f_{i,\boldsymbol{\theta}_0^{(i)}}(X_{i,0,T})} \pi\left(\boldsymbol{\theta}_1^{(i)}\right) d\boldsymbol{\theta}_1^{(i)} \qquad (5.5)$$

for $j = 1, \cdots, 100$, where $n = 15$, $T = 5$ and $\boldsymbol{\theta}_0^{(i)}$ is the true parameter set corresponding to the *i*-th individual.

We obtain the *MLE* of the 15 parameter sets by simulated annealing. Then we calculate the Bayes factor with the prior such that the parameter components are independent normal with means as the respective *MLE*s and variances 1. In all the cases corresponding to 100 covariate combinations we obtain $\frac{1}{nT} log I_{nT}^{j} < 0$ for $j = 1, \cdots, 100$. Thus, Bayes factor indicated the correct covariate combination in all the cases considered. We also considered the case when a normal prior is considered for the parameters of the true model. In this case with respect to the component-wise independent normal prior with individual mean as obtained from simulated annealing and component-wise variance 0.1^2, we obtain

$$\frac{1}{15 \times 5}\left[\log\left(\prod_{i=1}^{15}\int f_{i,\boldsymbol{\theta}_1^{(i)}}^{j}(X_{i,0,T})\pi(\boldsymbol{\theta}_1^{(i)})d\boldsymbol{\theta}_1^{(i)}\right) - \log\left(\prod_{i=1}^{15}\int f_{i,\boldsymbol{\theta}_0^{(i)}}(X_{i,0,T})\pi(\boldsymbol{\theta}_0^{(i)})d\boldsymbol{\theta}_0^{(i)}\right)\right] < 0,$$

for $j = 1, \cdots, 100$. Indeed, it turned out that

$$\tfrac{1}{15\times 5}\log\left(\prod_{i=1}^{15}\int f_{i,\boldsymbol{\theta}_0^{(i)}}(X_{i,0,T})\pi(\boldsymbol{\theta}_0^{(i)})d\boldsymbol{\theta}_0^{(i)}\right) = 0.4865$$

and the maximum of $\tfrac{1}{15\times 5}\log\left(\prod_{i=1}^{15}\int f_{i,\boldsymbol{\theta}_1^{(i)}}^{j}(X_{i,0,T})\pi(\boldsymbol{\theta}^{(i)})d\boldsymbol{\theta}^{(i)}\right)$ is 0.4127. In other words, the Bayes factor consistently selects the correct model even in this situation.

5.3 Company-wise data from national stock exchange

To deal with real data we collect the stock market data (467 observations during the time range August 5, 2013, to June 30, 2015) for 15 companies which is available on *www.nseindia.com*. The nature of some company-wise data are shown in Figure 1.

Each company-wise data is modeled by various availabe standard financial *SDE* models with the available "fitsde" package in *R*. After obtaining the BIC (Bayesian Information Criterion) for each company corresponding to each available financial model, we find that the minimum value of BIC corresponds to the *CKLS* model, given, for process $X(t)$, by

$$dX(t) = (\theta_1 + \theta_2 X(t))dt + \theta_3 X(t)^{\theta_4} dW(t).$$

As per our theory we treat the diffusion coefficient as a fixed quantity. So, after obtaining the estimated value of the coefficients by the "fitsde" function, we fix the values of θ_3 and θ_4, so that the diffusion coefficient becomes fixed. We let $\theta_3 = A$, $\theta_4 = B$.

In this *CKLS* model, we now wish to include time varying covariates. In our work we consider the "close price" of each company. The stock market data is assumed to be dependent on IIP general index, bank interest rate, US dollar exchange rate and on various other quantities. But we assume only these

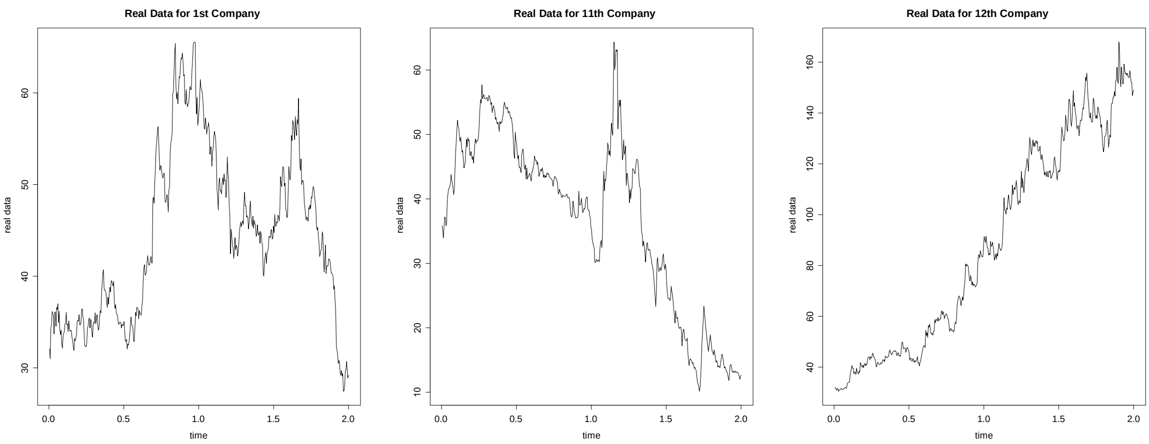

Figure 1: Some company-wise time-series data.

three quantities as possibly the most important time dependent covariates.

Briefly, IIP, that is, index of industrial production, is a measurement which represents the status of production in the industrial sector for a given period of time compared to a reference period of time. It is one of the best statistical data, which helps us measure the level of industrial activity in Indian economy. Its importance lies in the fact that low industrial production will result in lower corporate sales and profits, which will directly affect stock prices. So a direct impact of weak IIP data is a sudden fall in stock prices.

As the IIP data is purely industrial data, banking sector is not included in it. So, we also consider the bank interest rate as another covariate. Note that, higher the bank interest rate, fixed deposits become more attractive and one will preferably deposit money in bank rather than invest in stock market. Besides, companies with a high amount of loans in their balance sheets would be affected very seriously. Interest cost on existing debt would go up affecting their EPS (Earning per Share) and ultimately the stock prices. But during low interest rate these companies would stand to gain. Banking sector is likely to benefit most due to high interest rates. The Net Interest Margins (it is the difference between the interest they earn on the money they lend and the interest they pay to the depositors) for banks is likely to increase leading to growth in profits and the stock prices. Hence, it is clear that, the interest rates and stock markets are inversely related. As the interest rates go up, stock market activities tend to come down.

Finally, exchange rates directly affect the realized return on an investment portfolio with overseas holdings. If one own stock in a foreign company and the local currency goes up, the value of the investment also goes up. Foreign investment is also related very much to US dollar exchange rate.

Hence, we collect the values of the aforementioned time varying covariates during the time range August 5, 2013, to June 30, 2015. The pattern of the covariates are displayed in Figure 2.

We denote these three covariates by c_1, c_2, c_3 respectively. Now, our considered *SDE* models for national stock exchange data associated with the 15 companies are the following:

$$dX_i(t) = (\theta_1^i + \theta_2^i c_1(t) + \theta_3^i c_2(t) + \theta_4^i c_3(t))(\theta_5^i + \theta_6^i X_i(t))dt + A^i X_i(t)^{B^i} dW_i(t),$$

for $i = 1, \cdots, 15$.

5.3.1 Selection of covariates by Bayes factor

Among the considered three time varying covariates we now select the best set of covariate combinations for the 15 companies among 100 such sets through Bayes factor, computing the log-marginal-likelihoods with respect to the normal prior on the parameter set, assuming *a priori* independence of the parameter components with individual means being the corresponding *MLE* (based on simulated annealing) and

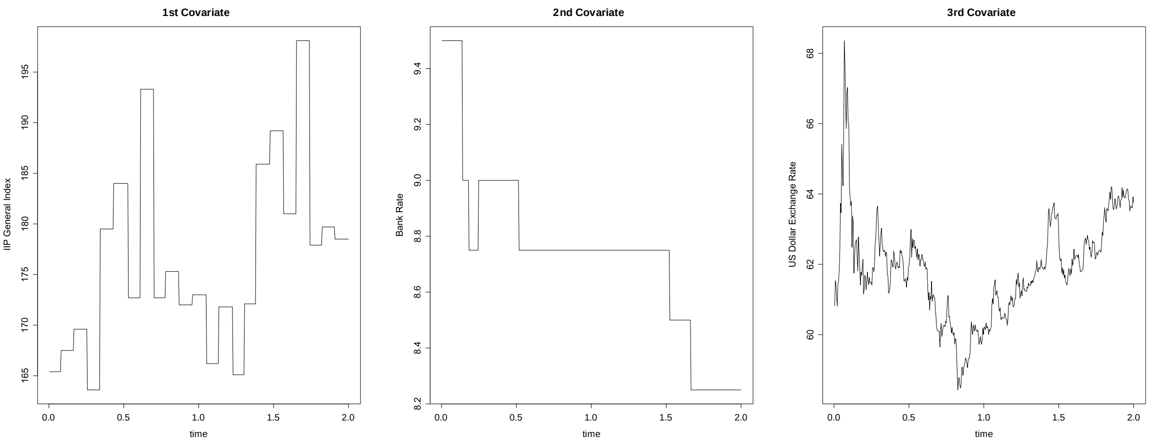

Figure 2: Covariates.

0.01^2 variance (relatively small variance ensured numerical stability of th marginal likelihood). Table 3 provides the sets of covariates for the 15 companies obtained by our Bayes factor analysis. Also observe that each of the three covariates occurs about 50% times among the companies, demonstrating that overall impact of these on national stock exchange is undeniable.

Table 3: Company-wise covariates obtained by Bayes factor analysis

Company	Covariates
1	Bank rate
2	US dollar exchange rate
3	None
4	None
5	Bank rate and US dollar exchange rate
6	Bank rate and US dollar exchange rate

7	IIP general index and US dollar exchange rate
8	Bank rate
9	IIP general index and Bank rate
10	IIP general index
11	IIP general index, Bank rate and US dollar exchange rate
12	IIP general index and Bank rate
13	US dollar exchange rate
14	IIP general index, Bank rate and US dollar exchange rate
15	IIP general index

Summary and discussion

This article establishes the asymptotic theory of Bayes factors when the models to be compared are systems of *SDE*'s consisting of time-dependent covariates and random effects, assuming that the number of individuals as well as the domains of observations of the individuals increase indefinitely. Different initial values for different *SDE*'s are also allowed. The only instance of related effort in this direction is that of Maitra and Bhattacharya (2018). The main difference of our undertaking with that of Maitra and Bhattacharya (2018) is that they assumed the domains of observations to be fixed for the individuals, a consequence being that incorporation of random effects in their model was not possible from the asymptotic perspective. Moreover, in their case, a single set of covariates was associated with all the individuals, but here our random effects set-up allows different sets of time-dependent covariates for different individuals.

To proceed, we first needed to build an asymptotic theory of Bayes factors for comparing two individual *SDE*'s, rather than two systems of *SDE*'s, as the domain of observation expands. Our results in this regard, which help formulate our asymptotic theory for comparing two systems of *SDE*'s using Bayes factors, are perhaps also of independent interest, being possibly the first ever results in this direction of research. Although the relevant variance did not converge to zero when two individual *SDE*'s are compared, we are able to establish almost sure exponential convergence of the Bayes factor when the number of subjects are allowed to increase indefinitely. Importantly, our theory covers both *iid* and non-*iid* cases.

Our simulation studies associated with covariate selection demonstrate that Bayes factor yields consistent results even in non-asymptotic situations. Bayes factor analysis of a real data on company-wise national stock exchange also yielded plausible sets of covariates for the companies.

Note that our current asymptotic Bayes factor theory remains valid for comparison between *iid* and non-*iid* models. For instance, if the true model consititutes an *iid* system, then $f_{0i} \equiv f_0 \equiv f_{\theta 0}$; the rest remains the same as the theory for our non-*iid* setting. The situation is analogous when the other model forms an *iid* system.

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References

[1] Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In B. N. Petrov and F. Csaki, editors, *Second International Symposium on Information Theory*, pages 267–281, Budapest. Academiai Kiado. Reprinted in S. Kotz and N. L. Johnson (Eds) (1992). *Breakthroughs in Statistics Volume I: Foundations and Basic Theory*, pp. 610–624. Springer-Verlag.

[2] Claeskens, G. and Hjort, N. L. (2008). *Model Selection and Model Averaging*. Cambridge University Press, Cambridge, UK.

[3] Delattre, M., Genon-Catalot, V., and Samson, A. (2013). Maximum Likelihood Estimation for Stochastic Differential Equations with Random Effects. *Scandinavian Journal of Statistics*, 40, 322–343.

[4] Dey, D. K., Ghosh, S. K., and Mallick, B. K. (2000). Generalized Linear Models: A Bayesian Perspective.

[5] Iacus, S. M. (2008). Simulation and Inference for Stochastic Differential Equations: With R Examples. Springer, New York.

[6] Jiang, J. (2007). Linear and Generalized Linear Mixed Models and Their Applications. Springer, New York, USA.

[7] Kass, R. E. and Raftery, R. E. (1995). Bayes factors. *Journal of the American Statistical Association*, 90(430), 773–795.

[8] Kontoyiannis, I. and Meyn, S. P. (2003). Spectral Theory and Limit Theorems for Geometrically Ergodic Markov Processes. *The Annals of Applied Probability*, 13, 304–362.

[9] Leander, J., Almquist, J., Ahlstrom, C., Gabrielsson, J., and Jirstrand, M. (2015). Mixed Effects Mod-¨ eling Using Stochastic Differential Equations: Illustrated by Pharmacokinetic Data of Nicotinic Acid in Obese Zucker Rats. *The AAPS Journal*, 17, 586–596.

[10] Maitra, T. and Bhattacharya, S. (2015). On Bayesian Asymptotics in Stochastic Differential Equations with Random Effects. *Statistics and Probability Letters*, 103, 148–159. Also available at "http://arxiv.org/abs/1407.3971".

[11] Maitra, T. and Bhattacharya, S. (2016). On Asymptotics Related to Classical Inference in Stochastic Differential Equations with Random Effects. *Statistics and Probability Letters*, 110, 278–288. Also available at "http://arxiv.org/abs/1407.3968".

[12] Maitra, T. and Bhattacharya, S. (2018). Asymptotic Theory of Bayes Factor in Stochastic Differential Equations: Part I. ArXiv Preprint.

[13] Mao, X. (2011). *Stochastic Differential Equations and Applications*. Woodhead Publishing India Private Limited, New Delhi, India.

[14] Muller, A., Scealy, J. L., and Welsh, A. H. (2013). Model Selection in Linear Mixed Models.¨ *Statistical Science*, 28, 135–167.

[15] Oravecz, Z., Tuerlinckx, F., and Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. *Psychological Methods*, 16, 468–490.

[16] Overgaard, R. V., Jonsson, N., Tornœ, C. W., and Madsen, H. (2005). Non-Linear Mixed-Effects Models with Stochastic Differential Equations: Implementation of an

Estimation Algorithm. *Journal of Pharmacokinetics and Pharmacodynamics*, 32, 85–107.

[17] Roberts, G. and Stramer, O. (2001). On Inference for Partially Observed Nonlinear Diffusion Models Using the Metropolis-hastings Algorithm. *Biometrika*, 88, 603–621.

[18] Schwarz, G. (1978). Estimating the Dimension of a Model. *The Annals of Statistics*, 6, 461–464.

[19] Sivaganesan, S. and Lingham, R. T. (2002). On the Asymptotic of the Intrinsic and Fractional Bayes Factors for Testing Some Diffusion Models. *Annals of the Institute of Statistical Mathematics*, 54, 500–516.

Commonality and Differences of the B2B sales process in emerging market: A qualitative exploration

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Abstract

According to the past researchers, socio-economic and ecological vulnerability are some of the characteristics of B2B organizations operating in emerging markets. Hence creating well-researched propositions related to improving both efficiency and effectiveness of the B2B sales process will help the organizations in survival and sustainable growth. This intern will result in retaining corporate reputation in terms of both market and financial value.

The reports published by renowned research organizations like Government of science Foresight indicate that the current global textile market constitutes around 2 percent of the world's GDP and countries like EU, USA and China are the world's largest textile markets. The combined share of these markets is about 54 percent. Further, the estimated CAGR of the textile market for the BRIC (Brazil – 5%, Russia – 3%, India – 12%, China – 10% by 2025) indicates the enhancing role of emerging market for this sector. The similar trend is observed for the pharmaceutical market operating in the BRICS (Brazil, \$18.3B; Russia, \$10.7B; India, \$13.9B; China, \$75.7B, and South Africa, \$2.5B). According to the Shanghai Academy of Social Sciences, the e-commerce sector has witnessed a strong growth with a rate of 20 percent specifically in BRICS's. This growth rate is far above the global average. These facts indicate the potential of the emerging market for the e-commerce sector. The global statistics of e-commerce transactions indicate that business to business (B2B) transactions contribute up to 90 percent which indicates the relevance of B2B sector.

Based on these facts and scholarly contributions of past researcher, the researcher realized the scope of the B2B sector as an emerging research avenue. Further, it is

observed that in case of emerging markets the major emphasis of the marketing researchers is on B2C (Business-to-Consumer). The volume driven nature of the B2C may be one of the reasons for this trend. These facts also indicate the research scope in the emerging markets for studies focusing on B2B.

In line with the aforementioned information, the main objective of this study is to understand the contextual commonalities & differences of the business to business (B2B) sales process with special reference to the textile, pharmaceutical, and e-commerce in emerging markets. The past researchers reported that predominantly the B2B sales process is standardized in nature still the contextual dimensions of B2B domain like role of lead pool, role of innovation, role of buying center, role of market information and market cycle, role of technical knowledge and product complexity, role of business relationships and the scale of orders differ from industry to industry. This indicates that the commonalities related to sales practices in B2B are well researched and created a scope to have knowledge about contextual differences with special reference to emerging markets.

In order to gain insights and formulate the sector-specific propositions, the theoretical perspectives will be understood through a systematic review of literature emphasizing on the various factor related to the sales process and the context. The focus of this literature review will be on B2B sales processes deployed in emerging markets. In order to gain a practitioner's perspective focus group discussions would be conducted to develop propositions.

Keywords – B2B, Industrial Marketing, Sales Process, Emerging Markets, Sustainable Growth

Is India's "Green" path too steep?

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Abstract

After 20 years of Kyoto protocol there is remarkable awareness and certain measures to mitigate the climate change are developed. In December 2016, nearly 194 countries signed the Paris Agreement and promised to aggressively cut the Green House Gas (GHG) emissions within the given timeframe.

Theoretically, climate is not a public good only but rather a Global Public Good. Once the climate change occurs, a specific country can neither choose to live in the climate of the past nor can its costs of adjustments to climate change borne be in limits as to the costs borne by other countries. With this, the Free Rider[1] problem enters and to overcome it and increase the social welfare, some mechanism to ensure a clean environment becomes essential. The costs of environmental damages are externalized on the whole world. So putting a price on GHG emissions is a way to internalize these costs. Here the role of a fiscal policy gets underlined.

There are two effective ways in which a country can reduce the GHG emissions, first is to go for renewable energy sources, more environment friendly ways to utilise energy for different production processes and thereby contribute to mitigate the climate change. Secondly, a country can impose a carbon tax on petroleum products, coal and other fuels to curtail their usage and then obviously the GHG emission as well. Imposing a tax would obviously mean discouraging the usages of the taxed commodity, however may result in lower production of goods as a threat.

Based on the empirical data this paper conceptualises India's case for the same where India's proactive efforts on environmental activism are showcased, at the same time it also brings out the efficacy of imposing a carbon tax vis a vis alternatives to reduce the carbon emissions as Mitigation and Adaptation, those are implied under the Clean Development Mechanism (CDM) activities under the Kyoto Framework. This has greater implications for India as a trade off between its sustainable development needs and the climate change externalities. India is a developing country with increasingly vast energy needs. The taxing of emissions discourages the consumption of fossil fuels by making the production processes more costly. This creates a conflict while choosing between fuelling development and saving the environment. The paper thus brings out the intricacies in the Green Path of India.

Keywords: Sustainable development, Carbon tax, CDM, Mitigation, Abatement cost

A multilevel analysis of Job Demands and Intention to Resign through Perceived Service Recovery Performance

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Abstract

The present study investigates the influence of job demands on intention to resign of front line employees (FLEs) of courier industry. Moreover, it also focuses on neglected area of perceived service recovery performance which is the fundamental aspect of success in services industry. The current study incorporates perceived service recovery performance as mediator between job demands and intention to resign under the light of cognitive dissonance and social exchange theory. Most of the past studies have focused on banking industry, food industry, and transport and tourism industry. The courier industry is neglected in past studies regarding front line employees. A random sampling of 700 front line employees of courier industry was selected. The results reflected that higher job demands results in intention to resign. Whereas, perceived service recovery performance mediates the relationship between elements of job demands and intention to resign. Researchers have also discussed the limitation and direction for future research.

Key words: Job demands, role conflict, role over load, role ambiguity, intention to resign, perceived service recovery performance, courier industry, front line employees

Classification of Dynkin Diagrams And Root Systems Of Indefinite Quasi Hyperbolic Kac Moody Algebras Of Rank 6

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ABSTRACT

Kac Moody algebras is one of the modern fields of mathematical research which has got wide applications to various branches of Mathematics and Mathematical Physics, like quantum mechanics, non-linear systems, differentiable manifolds, topology, geometry etc. Among the three main classes of Kac Moody algebras, namely finite, affine and indefinite classes, a lot of work had already been done in the case of finite and affine types. The in depth structure of indefinite Kac Moody algebras is yet to be explored completely.

In this work, a class of indefinite, non-hyperbolic type, namely the quasi hyperbolic Kac Moody algebras and the corresponding Dynkin diagrams of rank 6 are considered. Complete classification of Dynkin diagrams of rank 6 quasi hyperbolic Kac Moody algebras is obtained. Classification of Dynkin diagrams will equivalently give a classification of the associated Generalized Cartan Matrices(GCM). Only symmetrizable GCM's will be considered , since the existence of the symmetric, invariant, non degenerate bilinear form defined on the roots and co-roots are guaranteed only for symmetrizable GCM.

Root systems are the essential components in understanding the structure of these algebras. Some of the basic properties of roots like, Isotropic roots, purely imaginary roots , minimal imaginary roots etc. for these families of algebras are studied; Examples are given for the existence of these roots. Only Symmetrizable GCM's will be considered , since the existence of the symmetric, invariant, non-degenerate bilinear form defined on the roots and co-roots exists only when the GCM is symmetrizable.

A realization for a specific class in rank 6 quasi hyperbolic family is considered. These algebras are first realized as graded Lie algebras of Kac Moody type; Using the homological and spectral sequences theory, the homology modules (upto level 3)

are computed. The structure of some specific families in this class of quasi hyperbolic algebras is understood through the graded components. Representation theory is invoked for obtaining the realization for quasi hyperbolic Kac Moody algebras.

INTRODUCTION

Kac Moody algebras developed independently and simultaneously by Victor G Kac and Robert Moody [8] is an interesting branch of Mathematics, which has got significant applications in different branches of Mathematics and Mathematical Physics like conformal field theory, theory of exactly solvable models, quantum mechanics, geometry, etc. Among the broad classification of Kac Moody algebras namely finite, affine and indefinite types, lot of study remains to be undertaken in the indefinite category;

Sthanumoorthy and Uma Maheswari [14] introduced a new class of roots called purely imaginary roots and a new class of indefinite non hyperbolic Kac Moody algebras namely extended hyperbolic type, which satisfy the purely imaginary property; Hechun Zhang and Kaiming Zhao in [7] defined quasi finite GCM and studied the properties of imaginary roots. Feingold and Frenkel [6] computed level 2 root multiplicities for $HA^{(1)}_1$ and $HA^{(1)}_n$. Kang ([9], [10], [11],[12]) determined the structure and root multiplicities for roots upto level 5 for $HA^{(1)}_1$, $HA^{(1)}_2$ and $HA^{(1)}_n$. Benkart et.al ([1], [2],[3]) studied Indefinite Kac-Moody algebras of special linear type and classical type. Strictly imaginary roots and special imaginary roots were studied by Casperson [5] and Bennett [4].

Extended hyperbolic Kac-Moody algebras $EHA_1^{(1)}$ and $EHA_2^{(2)}$([14], [15] , [16] [17] ,

[18]) root multiplicities of some extended-hyperbolic Kac-Moody and generalized Kac-Moody algebras were studied . In ([19] Uma Maheswari introduced the quasi affine type of Dynkin diagramsand studied the quasi affine family $QAG^{(1)}_2$; Indefinite quasi affine Kac Moody algebras QHG_2, $QHA_2^{(1)}$, $QAC_2^{(1)}$ and $QAD^{(2)}_3$. Uma Maheswari introduced another new class of Dynkin diagrams and associated Kac Moody algebras of quasi hyperbolic type in ([20], [21] , [22], [23], [24], [25], [26]); Rank 3 Dynkin diagrams of quasi hyperbolic Kac Moody algebras were classified and properties of roots were studied.

In this paper, the basic definitions and concepts needed for further study is recalled in Section 2; In Section 3,the complete classification of connected, non-isomorphic Dynkin diagrams associated with the Generalized Cartan Matrices of indefinite, Quasi hyperbolic Kac Moody algebras of rank 6 is obtained. In Section 4, properties of imaginary roots, for specific families' rank 6 are studied. A realization for a class of indefinite quasi hyperbolic Kac Moody algebras of rank 6 is obtained as a graded Lie algebra of Kac Moody type. For this, the representation theory of infinite dimensional Lie algebras is invoked. The homological components are computed using the spectral sequences theoretical approach.

PRELIMINARIES

In this section , basic definitions and results required for the study are given. For a detailed study, one can refer [8], [27].

Definition 2.1[27]. Let A be a GCM $A=(a_{ij})^{n}_{i,j=1}$, rank of A= l, $(H, \prod, \prod^{v})$ be a realization of A,$H = 2n - l$ dimensional complex vector space (where), $\prod = \{a_1, \ldots, a_n\} \subset H^*$ be the set of simple roots and $\prod^{v} = \{a^{v}_1, \ldots, a^{v}_n\} \subset H$ the set of simple co roots, satisfying $a_j (a^{v}_i) = a_{ij}$ for $i, j = 1, \ldots, n$.

Definition 2.2[22]. Let $A=(a_{ij})^{n}_{i,j=1}$ be a symmetrizable GCM. The Kac-Moody algebra $g(A)$ is the Lie algebra over C generated by the elements e_i , f_i , $i = 1, 2, \ldots, n$ and h satisfying the following relations:

$[h, h] = 0, h, h \in H$; $[e_i , f_j] = \delta_{ij} a_j^{v}$;

$[h, e_j] = a_j (h)e_j$; $[h, f_j] = -a_j (h)f_j$, $i, j \in N$; $(ad e_i)^{1-a_{ij}} e_j = 0$;

$(ad f_i)^{1-a_{ij}} f_j = 0, \forall i \neq j, i,j \in N$

Root space Decomposition: $g(A) = \oplus g_{\alpha}(A), a \in Q$

where $g_a(A) = \{x \in g(A) / [h, x] = a(h)x$, for all $h \in H\}$. An element a, a 0 in Q is called a root if g_a =0.

Let $\Delta (= \Delta (A))$ denote the set of all roots of $g(A)$ and Δ_+, the set of all positive roots of $g(A)$, $\Delta_- = - \Delta_+$ and $\Delta = \Delta_+ \cup \Delta_-$.

A positive imaginary root α is said to be purely imaginary if for any positive imaginary root β, $\alpha+\beta$ is imaginary[9]. The associated Kac Moody algebra satisfies the purely imaginary property if every imaginary root is purely imaginary.

A root α is strictly imaginary if for any real root β, either $\alpha+\beta$ or $\alpha-\beta$ is root [2].

Definition 2.3. [22] To every GCM A is associated a Dynkin diagram $S(A)$ defined as follows: $S(A)$ has n vertices and vertices i and j are connected by max $\{|a_{ij}|, |a_{ji}|\}$ number of lines if $a_{ij} . a_{ji} \leq 4$ and there is an arrow pointing towards i if $|a_{ij}| > 1$. If $a_{ij} . a_{ji} > 4$, i and j are connected by a bold faced edge, equipped with the ordered pair

$(|a_{ij}|, |a_{ji}|)$ of integers.

Definition 2.4. [22] A Kac-Moody algebra $g(A)$ is said to be of finite, affine or indefinite type if the associated GCM A is of finite, affine or indefinite type respectively.

Definition 2.5. [12] Let $A = (a_{ij})^n_{i,j=1}$ be an indecomposable GCM of indefinite type. We define the associated Dynkin diagram $S(A)$ to be of Quasi-hyperbolic (QH) type if $S(A)$ has a proper connected sub diagram of hyperbolic types with $n - 1$ vertices. The GCM A is of Quasi-hyperbolic type if $S(A)$ is of Quasi-hyperbolic type. We then say the Kac-Moody algebra $g(A)$ is of Quasi-hyperbolic type.

GENERAL CONSTRUCTION OF GRADED LIE ALGEBRA [1]

The general construction of graded Lie algebras of Kac Moody type is briefly given here. For more details one can refer [1].

Let us use the following notations :

G – a Lie algebra over a field of characteristic zero.

– basic representation of highest weight module over G

*– contragradiant of V

: $V \otimes V^* \to G$ a G-module homomorphism.

Then $V \oplus G \oplus V^*$ has a local Lie algebra structure and extending the the bracket operation, $G = \sum G_n$ becomes a graded Lie algebra structure generated by the local part. For $n \geq 1$, define the subspaces,

$I_{\pm n} = \{x \in G_{\pm}/[y_1, [\ldots [y_{n-1}, x]] \ldots] = 0$ for all $y_1, \ldots, y_{n-1} \in G_{\mp}\}$.

Set $I_+ = \sum_{n>1} I_n$, $I_- = \sum_{n>1} I_{-n}$,

where I_+ and I_- *are* maximal graded ideals of G trivially intersecting the local part $G_{-1}+G_0+G_1$. For $n > 1$, let $L_{\pm n} = G_{\pm n}/I_{\pm n}$. Let $L = L(G, V, V, \psi) = G_-/I_- \oplus G_0 \oplus G_+/I_+ = \cdots \oplus L_{-2} \oplus L_{-1} \oplus L_0 \oplus L_1 \oplus L_2 \oplus \ldots$, where $L_0 = G_0$, $L_1 = G_1$, $L_{-1} = G_{-1}$.

Thus $L = G/I$, where $G = \sum_{n=-\infty}^{n=\infty} G_n$ and $I = I_+ + I_-$ is the largest graded ideal trivially intersecting the local part. Also L is a graded Lie algebra generated by the subspace $L_1 = G_1$.

{ We recall the general construction of graded Lie algebra.
Let G be a Lie algebra over a field of characteristic zero. Let V, V' be two G – modules and
$\psi: V' \otimes V \to G$ be a G – module homomorphism. Consider $G_0 = G, G_{-1} = V, G_1 = V'$.
$G_+ = \sum_{n\geq1} G_n$ (resp. $G_- = \sum_{n\geq1} G_{-n}$) denote the free Lie algebra generated by V' (resp. V).
G_n (resp G_{-n}) for n > 1 is the space of all products of n vectors from V' (resp. V)
$G = \sum_{n=-\infty}^{\infty} G_n$ is given a Lie algebra structure by defining the Lie bracket [,] as follows:
(i). [a, v] = a.v = –[v, a] and [a, w] = a.w = –[w, a] $\forall$ a, b $\in$ G, $v \in V$, $w \in V'$
(ii). [a, b] denote the bracket operation in G $\forall$ a, b $\in$ G.

(iii). $[w,v] = \psi(w \otimes v) = -[v,w], \forall$ v ∈V, w ∈ V'. $G = \sum_{n \in z} G_n$ becomes a graded Lie algebra which is generated by its local part $G_{-1} + G_0 + G_1$.

For n≥1 define the subspaces, $I_\pm = \{x \in G_\pm / [y_1,[\ldots[y_{n-1},x]]\ldots] = 0$ for all $y_1,\ldots y_{n-1} \in G_\mp\}$.
Set $I_+ = \sum_{n>1} I_n, I_- = \sum_{n>1} I_{-n}$ .Then I_+ and I_- are ideals of G and the ideal, I = $I_+ + I_-$ is the largest graded ideal of G trivially intersecting $G_{-1} + G_0 + G_1$.
For n >1, let $L_{\pm n} = G_{\pm n} / I_{\pm n}$.
Consider $L = L(G,V,V',\psi) = G\_/I\_ \oplus G_0 \oplus G_+/I_+ = \ldots \oplus L_{-2} \oplus L_{-1} \oplus L_0 \oplus L_1 \oplus L_2 \oplus \ldots$, where $L_0 = G_0, L_1 = G_1, L_{-1} = G_{-1}$.Then $L = \oplus_{n \in Z} L_n$ is a graded Lie algebra generated by its local part V ⊕ G ⊕ V' and L = G/I.
By the suitable choice of V (written as the direct sum of irreducible highest weight modules), the contragradient V^* of V, the basis elements and the homomorphism ψ : $V^* \otimes V \to g^e$, form the graded Lie algebra L = L(g^e, V, V^*, ψ) .

Theorem 2.4[1]: There is an isomorphism of g(A) – modules $H_j(L,J) \cong H_{j+2}(L)$, for j ≥ 1. In particular $I_{m+1} \cong (G_1 \otimes I_m) / H_3(L)_{m+1}$.

Now, for arbitrary j ≥ m, set $I^{(j)} = \sum_{n \geq j} I_n$; then $I^{(j)}$ is an ideal of G generated by the subspace I_j. We consider the quotient algebra $L^{(j)} = G/I^{(j)}$. Let $N^{(j)} = I^{(j)}/ I^{(j-1)}$. In this notation L = $L^{(m)}$. Then we have an important relation: $I_{j+1} \cong (G_1 \otimes I_j)/H_3(L^{(j)})_{j+1}$. And, there exists a spectral sequence $\{E^r_{pq}, d^r: E^r_{pq} \to E^r_{p-r,q+r-1}\}$ converging to $H_*(L^{(j)})$ such that $E^2_{p,q} \cong H_p(L^{(j-1)}) \otimes \wedge^q (I_{j-1})$ and $H_3(L^{(j)}) \cong E^\infty_{3,0} \oplus E^\infty_{2,1} \oplus E^\infty_{1,2} \oplus E^\infty_{0,3}$.

From the above notation, $H_2(L) \cong I_m$.

Theorem 2.5[9]: (Kostant's formula) $H_j(n^-(S), \tilde{V}(\lambda) = \bigoplus_{\substack{w \in W(S) \\ l(w)=j}} V(w(\lambda+\rho)-\rho)$.

Lemma 2.6[5]: Suppose $w = w' r_j$ and $l(w) = l(w')+1$. Then $w \in W(S)$ if and only if w' ∈ W(S) and $w'(\alpha_j) \in \Delta^+(S)$.

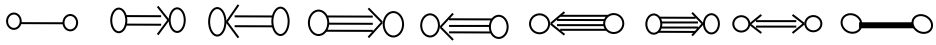
3. CLASSIFICATION OF DYNKIN DIAGRAMS OF RANK 6

In the notations of Wan[23], there are twenty two different families of hyperbolic Dynkin diagrams of rank 5, listed in Table Hyp.5[23].

Let us represent these 22 diagrams with the notations $H_1^{(5)}$, $H_2^{(5)}$, …, $H_{22}^{(5)}$.

Step 1: First let us consider $H_1^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_1^{(5)}$.

The added 6th vertex can be connected with the existing vertices of the hyperbolic diagram through any of the following nine edges:



…(***)

We shall consider the following cases separately.

Case 1) There is a single edge connecting a_6 with only one of the 5 vertices a_i, i=1,..,5

of $H_1^{(5)}$.

The vertex a_i can be chosen from the 5 vertices in $5C_1$ ways.

a_i and a_6, can be joined by one of the 9 possible edges

a_5  a_i

 can represent any of the nine possible edges given in (***).

Note that the bold faced edge has the labeling (b,b), where b > 2.

Then we get $9 \times 5C_1$ possible quasi hyperbolic Dynkin diagrams in this case. Excluding the isomorphic diagrams, we get 9x4 connected , non-isomorphic quasi hyperbolic Dynkin diagrams.

Case 2) a_6 is connected with any two of the 5 vertices a_i and a_j i,j=1,..,5 of $H_1^{(5)}$.

The vertices a_i and a_j can be chosen from the 5 vertices in $5C_2$ ways.

Each a_i and a_j can be joined with a_7, by one of the 9 possible edges listed above in (***).

Thus we get $9^2 \times 5C_2$ possible quasi hyperbolic Dynkin diagrams in this case.

Excluding the isomorphic diagrams, we get $9^2 \times 7$ connected , non-isomorphic quasi hyperbolic Dynkin diagrams.

Case 3) a_6 is connected with any three of the 5 vertices of $H_1^{(5)}$.

These three vertices can be chosen from the 5 vertices in $5C_3$ ways.

Each vertex a_j can be joined with a_6 by one of the 9 possible edges listed above in (***).

Thus we get 9^3 x$5C_3$ possible quasi hyperbolic Dynkin diagrams in this case.

Excluding the isomorphic diagrams, we get 9^3 x 6 connected , non-isomorphic quasi hyperbolic Dynkin diagrams.

Case 4) a_6 is connected with any four of the 5 vertices of $H_1^{(5)}$.

These four vertices can be chosen from the 5 vertices in $5C_4$ ways.

Each vertex a_j can be joined with a_6, by one of the 9 possible edges listed above in (***).

Thus we get 9^4 x$5C_4$ possible quasi hyperbolic Dynkin diagrams in this case. All these Dynkin diagrams are non isomorphic.

Case 5) a_6 is connected with all the five vertices of $H_1^{(5)}$.

Each vertex a_j can be joined with a_6, by one of the 9 possible edges listed above in (***).

Thus we get 9^5 possible quasi hyperbolic Dynkin diagrams in this case.

Step 2: Here let us consider $H_2^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_2^{(5)}$.

The added 6th vertex can be connected with the existing vertices of the hyperbolic diagram through any of the nine edges from (***).

We shall consider the following cases separately.

Case 1) There is a single edge connecting a_6 with only one of the 5 vertices a_i , $i=1,..,5$

of $H_2^{(5)}$.

The vertex a_i can be chosen from the 5 vertices in $5C_1$ ways.

a_i and a_6, can be joined by one of the 9 possible edges

 can represent any of the nine possible edges from (***):

Then we get $9 \times 5C_1$ possible quasi hyperbolic Dynkin diagrams in this case.

Excluding the isomorphic diagrams, we get 9 x 4 connected , non-isomorphic quasi hyperbolic Dynkin diagrams.

Case 2) a_6 is connected with any two of the 5 vertices a_i and a_j $i,j=1,..,5$ of $H_2^{(5)}$.

The vertices a_i and a_j can be chosen from the 5 vertices in $5C_2$ ways.

Each a_i and a_j can be joined with a_6, by one of the 9 possible edges listed above.

Thus we get $9^2 \times 5C_2$ possible quasi hyperbolic Dynkin diagrams in this case.

Excluding the isomorphic diagrams, we get $9^2 \times 7$ connected , non-isomorphic quasi hyperbolic Dynkin diagrams.

Case 3) a_6 is connected with any three of the 5 vertices of $H_2^{(5)}$.

These three vertices can be chosen from the 5 vertices in $5C_3$ ways.

Each vertex a_j can be joined with a_6 by one of the 9 possible edges listed above .

Thus we get 9^3 x5C_3 possible quasi hyperbolic Dynkin diagrams in this case.

Excluding the isomorphic diagrams, we get 9^3 x 6 connected , non-isomorphic quasi hyperbolic Dynkin diagrams.

Case 4) a_6 is connected with any four of the 5 vertices of $H_2^{(5)}$.

These four vertices can be chosen from the 5 vertices in 5C_4 ways.

Each vertex a_j can be joined with a_6, by one of the 9 possible edges listed above in (***).

Thus we get 9^4 x5C_4 possible quasi hyperbolic Dynkin diagrams in this case. All these Dynkin diagrams are non isomorphic.

Case 5) a_6 is connected with all the five vertices of $H_2^{(5)}$.

Each vertex a_j can be joined with a_6, by one of the 9 possible edges listed above in (***).

Thus we get 9^5 possible quasi hyperbolic Dynkin diagrams in this case.

Step 3: Here let us consider $H_3^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_3^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9\text{x}4 + 9^2\text{x}7 + 9^3\text{x}6 + 9^4\text{x}5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 4: Here let us consider $H_4^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_4^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x3 + 9^2x6 + 9^3x5 + 9^4x3 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 5: Here let us consider $H_5^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_5^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x3 + 9^2x6 + 9^3x5 + 9^4x3 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 6: Here let us consider $H_6^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_6^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x11 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 7: Here let us consider $H_7^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_7^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9\text{x}5 + 9^2\text{x}10 + 9^3\text{x}10 + 9^4\text{x}5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 8: Here let us consider $H_8^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_8^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9\text{x}3 + 9^2\text{x}7 + 9^3\text{x}6 + 9^4\text{x}3 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 9: Here let us consider $H_9^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_9^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9\text{x}4 + 9^2\text{x}7 + 9^3\text{x}7 + 9^4\text{x}2 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 10: Here let us consider $H_{10}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{10}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x4 + 9^2x7 + 9^3x7 + 9^4x2 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 11: Here let us consider $H_{11}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{11}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 12: Here let us consider $H_{12}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{12}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 13: Here let us consider $H_{13}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{13}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 14: Here let us consider $H_{14}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{14}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 15: Here let us consider $H_{15}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{15}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 16: Here let us consider $H_{16}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{16}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 17: Here let us consider $H_{17}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{17}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x7 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 18: Here let us consider $H_{18}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{18}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x7 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 19: Here let us consider $H_{19}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{19}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 20: Here let us consider $H_{20}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{20}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 21: Here let us consider $H_{21}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{21}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Step 22: Here let us consider $H_{22}^{(5)}$, extend this Dynkin diagram with one more vertex and represent the resulting diagram by $QH_{22}^{(5)}$.

As in the previous steps, by discussing the five cases where the 6th vertex is added to the existing vertices , connected with one, two, three, four and five edges, we can see that $\sum_{i=1}^{5}(9^i 5C_i)$ connected Dynkin diagrams of quasi hyperbolic types of rank 6 are obtained . Excluding the isomorphic diagrams we get $9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$ Dynkin diagrams of quasi hyperbolic type.

Summing up, the number of non isomorphic connected quasi hyperbolic Dynkin diagrams of rank 6 are

$$9x4 + 9^2x7 + 9^3x6 + 9^4x5 + 9^5 + 9x4 + 9^2x7 + 9^3x6 + 9^4x5 + 9^5 + 9x4 + 9^2x7 + 9^3x6 + 9^4x5 + 9^5 + 9x3 + 9^2x6 + 9^3x5 + 9^4x3 + 9^5 + 9x3 + 9^2x6 + 9^3x5 + 9^4x3 + 9^5 + 9x5 + 9^2x11 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x3 + 9^2x7 + 9^3x6 + 9^4x3 + 9^5 + 9x4 + 9^2x7 + 9^3x7 + 9^4x2 + 9^5 + 9x4 + 9^2x7 + 9^3x7 + 9^4x2 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x7 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x7 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$$

=2,095,308.

Theorem 3.1 (Classification Theorem) :

The total number of non-isomorphic connected Dynkin diagrams associated with the Quasi hyperbolic Kac Moody algebras of rank 6, is

$$9x4 + 9^2x7 + 9^3x6 + 9^4x5 + 9^5 + 9x4 + 9^2x7 + 9^3x6 + 9^4x5 + 9^5 + 9x4 + 9^2x7 + 9^3x6 + 9^4x5 + 9^5 + 9x3 + 9^2x6 + 9^3x5 + 9^4x3 + 9^5 + 9x3 + 9^2x6 + 9^3x5 + 9^4x3 + 9^5 + 9x5 + 9^2x11 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x3 + 9^2x7 + 9^3x6 + 9^4x3 + 9^5 + 9x4 + 9^2x7 + 9^3x7 + 9^4x2 + 9^5 + 9x4 + 9^2x7 + 9^3x7 + 9^4x2 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x7 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x7 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5 + 9x5 + 9^2x10 + 9^3x10 + 9^4x5 + 9^5$$

=2,095,308.

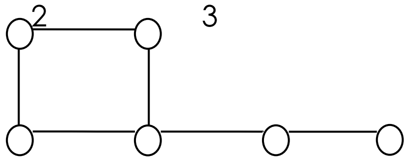
4 REALIZATION FOR $QH^{(6)}$

In this section, we are going to obtain a realization of a quasi hyperbolic Kac Moody algebra $QH^{(6)}$, whose associated Dynkin diagram is of of rank 6, as a graded Lie algebra of Kac Moody type.

The GCM associated with $QH^{(6)}$ is

$$C = \begin{pmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ -1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

The Dynkin diagram associated with $QH^{(6)}$ is 2 3

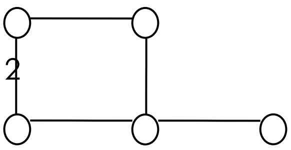


1 4 5 6

Construction as graded Lie algebra :

Let $A = \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$ be the hyperbolic GCM associated with the

1 2



Dynkin diagram

3 4 5

Let (H, Π, $\Pi^\vee$) be the realization of A with $\Pi = \{\alpha_1,\alpha_2,\alpha_3,\alpha_2,\alpha_4,\alpha_5\}$and $\Pi^\vee = \{\alpha_1^\vee,\alpha_2^\vee,\alpha_3^\vee,\alpha_4^\vee,\alpha_5^\vee\}$ representing the set of simple roots and simple co-roots respectively. Then we have the following bilinear relations :

Choose $\alpha_6^{'}$ in H* such that $(\alpha_6^{'},\alpha_5^{'}) = -1, (\alpha_6^{'},\alpha_6{'}) = 2$, and 0 otherwise

$(\alpha_1,\alpha_1)=2$, $(\alpha_1,\alpha_2)=-1$, $(\alpha_1,\alpha_3)=(\alpha_1,\alpha_5)=0;\ (\alpha_1,\alpha_4)=-1$,

$(\alpha_2,\alpha_2)=2, (\alpha_3,\alpha_3)=2, (\alpha_4,\alpha_4)=2, (\alpha_5,\alpha_5)=2$, $(\alpha_2,\alpha_3)=-1, (\alpha_2,\alpha_1)=-1$, ,

$(\alpha_3,\alpha_2)=-1$, $(\alpha_3,\alpha_4)=-1\ (\alpha_4,\alpha_5)=-1$

Define $\lambda=-2(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+\alpha_6'$ ε H *. Set $\alpha_6 = -\lambda$.

Let V be the integrable highest weight irreducible module over G with the highest weight λ and V* be its dual.

Let G be the hyperbolic Kac-Moody algebra associated with the GCM $\begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}$. Let ψ be the homomorphism between these modules and form the graded Lie algebra L(G^e, V, V*, ψ) as given in the general construction of graded Lie algebras in [1].

Let $<\alpha_i,\alpha_j> = \dfrac{2(\alpha_i,\alpha_j)}{(\alpha_i,\alpha_i)}, i,j=1,\ldots,6$. Form the matrix $C=(<\alpha_i,\alpha_j>)_{i,j=1}^{6}$. Then C =

$\begin{pmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ -1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$ is the symmetrizable, more specifically , a symmetric GCM of Quasi – hyperbolic type, the associated Kac-Moody algebra g(C) denoted by $QH^{(6)}$.

Then $L \cong g(C)$ and L is a symmetrizable Kac-Moody algebra of Quasi- hyperbolic type associated with the GCM C. Thus we have given the realization for this quasi hyperbolic family as a graded Lie algebra of Kac Moody type.

To understand the structure of these algebras, we study the graded components obtained in the realization, applying the spectral sequences and homological techniques developed by Benkart, et.al [1]. First, we compute the homology modules of the Kac-Moody algebra for $QH^{(6)}$. It suffices to study only about the negative part $L_- = G_- / I_-$.

Computation of Homology Modules

The only reflection of length 1 in W(S) is r_6.

$r_6(\rho) - \rho = -\alpha_6 \quad \therefore H_1(L_-) \cong V(-\alpha_6)$.

Reflections of length 2 in *W (S)*: r_6r_1, r_6r_2, r_6r_3, $r_6 r_4$, $r_6 r_5$,

$r_6r_1(\rho) - \rho = -\alpha_6 - \alpha_1$;

$r_6r_2(\rho) - \rho = -\alpha_6 - \alpha_2$;

$r_6r_3(\rho) - \rho = -\alpha_6 - \alpha_3$;

$r_6r_4(\rho) - \rho = -\alpha_6 - \alpha_4$;

$r_6r_5(\rho) - \rho = -2\alpha_6 - \alpha_5$;

Hence $H_2(L_-) \cong V(-\alpha_6 - \alpha_1) \oplus V(-\alpha_6 - \alpha_2) \oplus V(-\alpha_6 - \alpha_3) \oplus V(-\alpha_6 - \alpha_4) \oplus V(-2\alpha_6 - \alpha_5)$.

Reflections of length 3 in *W (S)*:

$r_6r_1r_2$, $r_6r_1r_3$ $r_6r_1r_4$, $r_6r_1r_5$ $r_6r_1r_2$ $r_6r_1r_6$ $r_6r_2r_1$ $r_6r_2r_3$,$r_6r_2r_4$, $r_6r_2r_5$,
$r_6r_2r_6$, $r_6r_3r_1$, $r_6r_3r_2$, $r_6r_3r_4$,

$r_6r_3r_5$, $r_6r_3r_6$, $r_6r_4r_1$, $r_6r_4r_2$, $r_6r_4r_3$, $r_6r_4r_5$, $r_6r_4r_6$,

$r_6r_4r_1$, $r_6r_4r_2$, $r_6r_4r_3$, $r_6r_4r_5$, $r_6r_4r_6$,

$r_6r_5r_1$, $r_6r_5r_2$, $r_6r_5r_3$, $r_6r_5r_4$, $r_6r_5r_6$.

$r_6r_1r_2(\rho) - \rho = -a_6 - a_2 - a_1$

$r_6r_1r_3(\rho) - \rho = -a_6 - a_3$

$r_6r_1r_4(\rho) - \rho = -a_6 - a_4$

$r_6r_1r_5(\rho) - \rho = -2a_6 - a_5$

$r_6r_2r_1(\rho) - \rho = -a_6 - a_2 - a_1$

$r_6r_2r_3(\rho) - \rho = -a_6 - a_2 - a_3$

$r_6r_2r_4(\rho) - \rho = -a_6 - a_4$

$r_6r_2r_5(\rho) - \rho = -2a_6 - a_5$

$r_6r_3r_1(\rho) - \rho = -a_6 - a_1$

$r_6r_3r_2(\rho) - \rho = -a_6 - a_3 - a_2$

$r_6r_3r_4(\rho) - \rho = -a_6 - a_4 - a_3$

$r_6r_3r_5(\rho) - \rho = -a_6 - a_1$

$r_6r_4r_1(\rho) - \rho = -a_6 - a_4 - a_1$

$r_6r_4r_2(\rho) - \rho = -a_6 - a_2$

$r_6r_4r_3(\rho) - \rho = -a_6 - a_4 - a_3$

$r_6r_4r_5(\rho) - \rho = -2a_6 - a_5 - a_4$

$r_6r_5r_1(\rho) - \rho = -a_6 - a_1$

$r_6r_5r_2(\rho) - \rho = -a_6 - a_2$

$r_6r_5r_3(\rho) - \rho = -a_6 - a_3$

$r_6r_5r_4(\rho) - \rho = -2a_6 - a_5 - a_4$

$r_6r_5r_6(\rho) - \rho = -a_6 - a_5$

Hence $H_3(L_-) \cong V(-a_6 - a_2 - a_1) \oplus V(-a_6 - a_3) \oplus V(-a_6 - a_4) \oplus$

$V(-2a_6 - a_5) \oplus V(-a_6 - a_2 - a_1) \oplus V(-a_6 - a_2 - a_3) \oplus$

$V(r_6r_2r_4(\rho) - \rho = -a_6 - a_4) \oplus V(-2a_6 - a_5) \oplus V(-a_6 - a_1) \oplus$

$V(-\alpha_6 - \alpha_3 - \alpha_2) \oplus V(-\alpha_6 - \alpha_4 - \alpha_3) \oplus V(-\alpha_6 - \alpha_1) \oplus$

$V(-\alpha_6 - \alpha_4 - \alpha_1) \oplus V(-\alpha_6 - \alpha_2) \oplus V(-\alpha_6 - \alpha_4 - \alpha_3) \oplus$

$V(-2\alpha_6 - \alpha_5 - \alpha_4) \oplus V(-\alpha_6 - \alpha_1) \oplus V(-\alpha_6 - \alpha_2) \oplus$

$V(-\alpha_6 - \alpha_3) \oplus V(-2\alpha_6 - \alpha_5 - \alpha_4) \oplus V(-\alpha_6 - \alpha_5)$.

Other homology components can be computed in a similar manner.

Properties of Roots:

Height two roots:

$(\alpha_1+\alpha_2,\alpha_1+\alpha_2)>0$,

$(\alpha_1+\alpha_4,\alpha_1+\alpha_4)>0$; $(\alpha_2+\alpha_3,\alpha_2+\alpha_3)>0$; $(\alpha_3+\alpha_4,\alpha_3+\alpha_4)>0$ $(\alpha_4+\alpha_5,\alpha_4+\alpha_5)>0$,

$(\alpha_5+\alpha_6,\alpha_5+\alpha_6)>0$

All height two roots are real.

Height three roots:

$(\alpha_1+\alpha_2+\alpha_3,\alpha_1+\alpha_2+\alpha_3)>0$, $(\alpha_1+\alpha_2+\alpha_4,\alpha_1+\alpha_2+\alpha_4)>0$

$(\alpha_1+\alpha_4+\alpha_5,\alpha_1+\alpha_4+\alpha_5)>0$, $(\alpha_2+\alpha_3+\alpha_4,\alpha_2+\alpha_3+\alpha_4)>0$

$(\alpha_3+\alpha_4+\alpha_5,\alpha_3+\alpha_4+\alpha_5)>0$, $(\alpha_4+\alpha_5+\alpha_6,\alpha_4+\alpha_5+\alpha_6)>0$

$(\alpha_1+2\alpha_2,\alpha_1+2\alpha_2)>0$, $(\alpha_1+2\alpha_4,\alpha_1+2\alpha_4)>0$; $(\alpha_2+2\alpha_3,\alpha_2+2\alpha_3)>0$;

$(\alpha_3+2\alpha_4,\alpha_3+2\alpha_4)>0$ $(\alpha_4+2\alpha_5,\alpha_4+2\alpha_5)>0$, $(\alpha_5+2\alpha_6,\alpha_5+2\alpha_6)>0$

$(2\alpha_1+\alpha_2,2\alpha_1+\alpha_2)>0$, $(2\alpha_1+\alpha_4,2\alpha_1+\alpha_4)>0$; $(2\alpha_2+\alpha_3,2\alpha_2+\alpha_3)>0$;

$(2\alpha_3+\alpha_4,2\alpha_3+\alpha_4)>0$ $(2\alpha_4+\alpha_5,2\alpha_4+\alpha_5)>0$, $(2\alpha_5+\alpha_6,2\alpha_5+\alpha_6)>0$

All height three roots are real.

Height four roots:

$$(\alpha_1+\alpha_2+\alpha_3+\alpha_4,\alpha_1+\alpha_2+\alpha_3+\alpha_4)=0$$

$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ is an imaginary root, in particular, isotropic root.

$$(\alpha_1+\alpha_2+2\alpha_4,\alpha_1+\alpha_2+2\alpha_4)>0$$

$$(2\alpha_1+\alpha_2+\alpha_4,2\alpha_1+\alpha_2+\alpha_4)>0$$

$$(\alpha_1+2\alpha_2+\alpha_4,\alpha_1+2\alpha_2+\alpha_4)>0$$

$$(\alpha_1+\alpha_2+2\alpha_4,\alpha_1+\alpha_2+2\alpha_4)>0$$

$$(\alpha_1+\alpha_2+2\alpha_3,\alpha_1+\alpha_2+2\alpha_3)>0$$

$$(\alpha_1+2\alpha_2+\alpha_3,\alpha_1+2\alpha_2+\alpha_3)>0$$

$$(2\alpha_1+\alpha_2+\alpha_3,2\alpha_1+\alpha_2+\alpha_3)>0$$

$$(\alpha_1+\alpha_4+2\alpha_5,\alpha_1+\alpha_4+2\alpha_5)>0,$$

$$(\alpha_1+2\alpha_4+\alpha_5,\alpha_1+2\alpha_4+\alpha_5)>0$$

$$(2\alpha_1+\alpha_4+\alpha_5,2\alpha_1+\alpha_4+2\alpha_5)>0$$

$$(\alpha_2+\alpha_3+2\alpha_4,\alpha_2+\alpha_3+2\alpha_4)>0$$

$$(\alpha_2+2\alpha_3+\alpha_4,\alpha_2+2\alpha_3+\alpha_4)>0$$

$$(2\alpha_2+\alpha_3+\alpha_4,2\alpha_2+\alpha_3+\alpha_4)>0$$

$$(\alpha_3+\alpha_4+2\alpha_5,\alpha_3+\alpha_4+2\alpha_5)>0,$$

$$(\alpha_3+2\alpha_4+\alpha_5,\alpha_3+2\alpha_4+\alpha_5)>0,$$

$$(2\alpha_3+\alpha_4+\alpha_5,2\alpha_3+\alpha_4+\alpha_5)>0,$$

$$(\alpha_4+\alpha_5+2\alpha_6,\alpha_4+\alpha_5+2\alpha_6)>0$$

$$(\alpha_4+2\alpha_5+\alpha_6,\alpha_4+2\alpha_5+\alpha_6)>0$$

$(2\alpha_4 + \alpha_5 + \alpha_6, 2\alpha_4 + \alpha_5 + \alpha_6) > 0$

$(\alpha_1 + \alpha_4 + \alpha_5 + \alpha_6, \alpha_1 + \alpha_4 + \alpha_5 + \alpha_6) > 0$, $(\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5, \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5) > 0$,

$(\alpha_2 + \alpha_1 + \alpha_4 + \alpha_5, \alpha_2 + \alpha_1 + \alpha_4 + \alpha_5) > 0$

$(\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6, \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6) > 0$

All the above roots are real.

The minimal imaginary root is $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$

CONCLUSION

The work done in this paper, helps us to understand the structure of the rank 6 Quasi-hyperbolic Kac-Moody algebras. Using the structure, we can further compute the multiplicities of roots, which would further lead to interesting applications using the representation theory.

REFERENCES

Benkart, G. M., Kang, S. J., and Misra, K. C., 1993, "Graded Lie algebras of Kac-Moody type," Adv. Math., 97, pp. 154–190.

Casperson, D., "Strictly imaginary roots of Kac Moody algebras", J. Algebra, Vol. 168,1994, pp. 90-122

Kac, V. G., 1990, "Infinite Dimensional Lie Algebra," 3rd ed. Cambridge : Cam-bridge University Press.

Kang, S. J., 1993, "Kac-Moody Lie algebras, spectral sequences and the Witt formula," Trans. Amer. Math. Soc., 339(2), pp. 463–493.

Kang, S. J., 1993, "Root multiplicities of the hyperbolic Kac-Moody Lie algebra $HA_1^{(1)}$," J. Algeba., 160, pp. 492–593.

Kang, S. J., 1994, "On the hyperbolic Kac-Moody Lie algebra $HA_1^{(1)}$," Trans. Amer. Math. Soc., 341, pp. 623–638.

Kang, S. J., 1994a, "Root multiplicities of the hyperbolic Kac-Moody Lie algebra $HA_n^{(1)}$," J. Algebra., 170, pp. 277–299.

Liu, L. S., 1992, "Kostant's formula for Kac-Moody Lie algebras," J. Algebra., 149, pp. 155–178.

Sthanumoorthy, N., and Uma Maheswari, A., 1996b, "Purely imaginary roots of Kac-Moody algebras," Comm. Algebra., 24(2), pp. 677–693.

Sthanumoorthy, N., and Uma Maheswari, A., 1996a, "Root multiplicities of ex-tended hyperbolic Kac-Moody algebras," Comm. Algebra., 24(14), pp. 4495–4512.

Sthanumoorthy, N., Lilly, P .L., and Uma Maheswari, A., 2004, "Root multiplicities of some classes of extended-hyperbolic Kac-Moody and extended - hyperbolic generalized Kac-Moody algebras," Contemporary Mathematics, AMS., 343, pp. 315–347.

Sthanumoorthy, N., Uma Maheswari, A., and Lilly. P .L., 2004, "Extended – Hy-perbolic Kac-Moody $\boldsymbol{EH\ A_2^{(2)}}$ Algebras structure and Root Multiplicities," Comm. Algebra., 32(6), pp. 2457–2476.

Sthanumoorthy, N., and Uma Maheswari, A., 2012, "Structure And Root Multi-plicities for Two classes of Extended Hyberbolic Kac-Moody Algebras $\boldsymbol{EH\ A_1^{(2)}}$ and $\boldsymbol{EH\ A_2^{(2)}}$ for all cases," Communications in Algebra., 40, pp. 632–665.

Uma Maheswari, A., 2011, A Study on Dynkin diagrams of Quasi hyperbolic Kac Moody algebras, Proceedings of the International Conference on Mathematics and Computer Science, ISBN 978-81-20490-0-7, pp. 115–119.

Uma Maheswari, A., 2014, "Quasi-Affine Kac-Moody Algebras: Classification of Dynkin Diagrams and Realization for a Class of $\boldsymbol{QAG_2^{(1)}}$," International Journal of Mathematical Science, 34(2), pp. 1590–1605.

Uma Maheswari, A., and Krishnaveni, S., 2014, "On the Structure of Indefinite Quasi-Hyperbolic Kac-Moody Algebras $\boldsymbol{QH\ A_2^{(1)}}$," International Journal of Alge-bra., 8(11), pp. 517–528.

Uma Maheswari, A., 2014, "A Study on the Structure of Indefinite Quasi-Affine Kac-Moody Algebras $\boldsymbol{QAC_2^{(1)}}$," International Mathematical Forum.,9(32), pp. 1595–1609.

Uma Maheswari, A., 2015, "Structure Of Indefinite Quasi-Affine Kac-Moody Al-gebras $\boldsymbol{QAD_3^{(2)}}$," International Journal of Pure and Applied Mathematics., 102(1), pp. 57–72.

Uma Maheswari, A., and Krishnaveni, S., 2014, "Structure of the Indefinite Quasi-Hyperbolic Kac-Moody Algebra ***QH*** $A_4^{(2)}$," International Mathematical Forum, HIKARI Ltd ., 9(31), pp. 1495–1507.

Uma Maheswari, A., and Krishnaveni, S., 2015, "A Study on the Structure of a class of Indefinite Non-Hyperbolic Kac-Moody Algebras ***QH*** $A_7^{(2)}$," International journal of Mathematical Sciences., 34(2), pp. 1639–1648.

Uma Maheswari, A., and Krishnaveni, S., 2015, "A Study on the Quasi-Hyperbolic Algebra ***QH*** $A_5^{(2)}$," International Journal of Pure and Applied Mathematics., 102(10), pp. 23–38

Wan Zhe-Xian., 1991, Introduction to Kac-Moody Algebra, Singapore: World Scientific Publishing Co. Pvt. Ltd.

Molecular and immunological diagnosis of *Toxoplasma gondii* and its relation to blood parameters in aborted women in Al-Basrah Governorate, Iraq.

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Abstract

Congenital Toxoplasmosis is one of the most serious forms of disease caused by a *Toxoplasma gondii* parasite, the effect of this parasite is not confined to the infection of the pregnant mother on the damage of physiological in her, but rather passes through the placenta to the fetus, resulting in severe damage ranging from miscarriage and congenital malformationswhich results in an incapacitated individual in the society who is a burden on his family. This study was conducted to determine the incidence of *Toxoplasma gondii* in aborted women who follow up toAl-Basrah Hospital for women and children in Al- Basrah governorate by using the serotype of the ELISA immunoglobulin test to detect IgM and IgG antibodies in the patient serum samples,this study included 180 blood samples for patients and 20 blood samples as control group for non-infected people for the period from May 2017 to November 2017. The results showed that the number of samples was 65/180 (36%), the samples that gave a positive result of antibodies IgG was 48 (73%), while that gave IgM 28 (33 %), this is evidence that the majority of infection to patients was in a previous period and that new infection is much less.

Molecular methods are the most recent, most accurate and sensitive in the diagnosis and have the ability to diagnose the modern infection depending on the gene B1 or B-30 for *Toxoplasma gondii* parasite, Real-Time PCR as the gold standard for the detection of parasite tetanus, the examination was conductedthe positive samples was28/180 (15.5%) in aborted women, the study of blood parameters of the patients showed a significant decrease in the concentration of hemoglobin in the red blood cells andthere was also an increase in white blood cells in the blood samples of infected women compared with the control group.

Keywords: ELISA, *T. gondii*, Congenital toxoplasmosis, Al-Basrah.

Introduction

Toxoplasmosis is a contagious disease that spreads all over the world, it is a serious disease common to humans and animals (Zoonotic disease) caused by a parasite belonging to the class Coccidia, known as *Toxoplasma gondii*, it is obligate intracellular parasite and has the ability to infect various tissues in many mammals and birds (1), because there is no specific host of the nonsexual stage in life cycle of this parasite, it can live them in all mammals, including humans and cats, While sexual stages occur Only in the intestines of the cats (2), the infection occurs to humans by eating raw meat or which are not well-cooked Containing tissue cysts, eating foods contaminated with egg cysts, drinking contaminated unpasteurized milk contaminated with Tachyzoites, dealing with soil contaminated with fecesof infected cats,and the Infection also occurs in the pericardial transition of the infected mother to the fetus as well as other transmission methods Such as organ transplantation or blood transfusion (3,4),the parasite can be passed through sexual contact in warm-blooded animals such as sheep, dogs and rats, but its transmission in humans has not been established yet (5). Transplantation of the parasite from the infected mother to the fetus through the placenta is the most serious because it causes Congenital malformationof the fetus when the mother acquires parasitic infection during pregnancy, but if a mother has parasitic infection before pregnancy, the likelihood of parasite transmission to the fetus is less frequent (6,7), The risk of this infection depends on the duration of pregnancy and the most serious of which is in the first months of pregnancy, where the stage of rapid reproduction Tachyzoite passes through the placenta in particular conditions before the formation of antibodies in the body of the pregnant mother and result in serious health problems ranging from miscarriage, hydrocephalus, intracerebral calcifications, retinochoroiditis, Hydrocephaly, retinitis, cardiomegaly, hepatosplenomegaly also may lead to death (7,8).The diagnosis of disease does not depend on the clinical symptoms, but through the examination of body fluids using by one of the tests of biological, serological, tissue or molecular methods (9).

The current study has relied on the use of one of theserological methods known as EnzymeLinked Immune Sorbent Assay (ELIZA), which can determine the presence of the parasite in the body through the antibodies IgG and IgM, wherethe presence of IgG antibodies indicates a previous infection with parasite, while IgM and IgG together indicate the presence of the disease at the time of the test (10), Sometimes serological tests fail to give real results because antibodies to the parasite do not form in the host's body until several weeks or the host's body fails to form it (11), to obtain more accurate results, molecular tests which are the latest technology are used to investigate the most accurate parasitic genetic material, the basis of these test is to detect DNA in body fluids and tissues and to amplify the B1 or P30 gene for parasitic DNA (12),Real Time PCR is the most accurate and sensitive in diagnosis and has proven to be effective in quantitative diagnosis of parasite (13), Which made us use this technique in our current study to give a realistic results of the incidence of parasitic infection,this study also aimed to clarify the relationship between the incidence of parasitic infection and the study of blood parameters for patients and compared with the control group healthy. The current study is considered the first of its kind in the province of Al-Basrah, where targeted methods of immunological and molecular diagnosis and the study of blood parameters in order to give a clear picture of the spread of parasite and its relationship with abortions of women in the province.

Material and Methods

A- Study area :

Al-Basrah:is an Iraqi city located in south of Iraq on the Shatt al-Arab between Kuwait and Iran. It is the third largest Governorate of Iraq in terms of 2.5 million in 2012, and the sixth largest Governorate in Iraq in terms of area (19,070 km2).· Basrah is also Iraq's main port, although it does not have deep water access,.it is consistently one of the hottest cities in Iraq, with summer temperatures regularly exceeding 50 °C (122 °F),http://www.basra.gov.iq/, (14) .

B- Collection of blood samples:

One hundred and eighty (180) blood samples were collected from 180 women surveyed to Basrah Hospital for women and children for the period from may 2017 to

October 2017 and suspected of toxoplasmosis,their ages ranged from (15-49) years With (8) ml of venous blood for abortion woman, the samples was divided as follows: (3 ml) of venous blood was isolated and the serum was extracted by using a centrifuge at 3000 cycles per minute for 10 minutes then stored in Eppendorf -tube at temperature -20C° until the time of ELISA testing (15), (2 ml) of venous blood put in a test tube containing an EDTA and store at -80 C° until DNA extraction (16), (3 ml) of venous blood put in a test tube containing an EDTA to study blood parameters (17).

C-samples examinations:

1- Enzyme-Linked Immune Sorbent Assay (ELIZA):

Use the ELISA test to detect IgM and IgG immunoglobulins in serum, This test depends on the interaction of the immune antibodies found in the serum of the infected person after dissolved and diluted with the high purity *T. gondii* antigens which is a sticker on the inner surface of the standard index plate drill (microwells) Thus making the immune complex (antibody-antigen complex), use for this purpose ELIZA kit produced by USA company (Biocheck) and were performed according to the manufacturer´s instructions.

2- Real-Time PCR methods:

DNA was extracted from the whole blood samples by using *ExiPrep*TM Plus Genomic DNA Kit from (Bioneer Company. Korea), performed according to the manufacturer´s instructions, after extraction DNA we went to Real-Time PCR (TaqMan probe) was performed for detection of *T.gondii* according to (19), we used Real-Time PCR TaqMan probe and primers for amplification of conserved region B1 gene in *T. gondii* when it found in samples DNA extraction of patients, the primers and probes performed according to (20) and it´s provided by (Bioneer Company. Korea), as in the following table (1):

Table (1): Real-Time PCR TaqMan probe and primers

primer	sequence		Product size
B1 primer	F	TCCCCTCTGCTGGCGAAAAGT	94bp
	R	AGCGTTCGTGGTCAACTATCGATTG	
B1 probe	5-6FAM-TCTGTGCAACTTTGGTGTATTCGCAG-TAMRA-3		

The Real-Time PCR amplification done by (AccuPower® DualStarTMqPCRPreMixBioneer. Korea), the qPCR master mix was prepared according to company instruction.

3- Blood parameters study:

Three (3 ml) of venous blood put in a test tube containing an EDTA inhibitor after placing it on the Shaker vibrator for 5 minutes and was used to study the blood parameters of the RBC to measure the hemoglobin ratioand the total and differential number of WBC by using the German-made CELL-DYN Ruby device 2012 is a system analyzer with a multi-scale automated with system designed for the Diagnostic tests in clinical laboratories (18), this system uses a flow cytometry technique where cells are examined by attaching them to a stream of reagents and passed through an electronic detector. The blood sample is placed in the tube below the pipette. The sample is then withdrawn by a special pipette. After about 40 seconds, the results of the blood parameters were read after processing the data by computer (17).

D- Statistical analysis:

The results were statistically analyzed according to (5.04), Graph pad software Inc. (USA) for (2010) by using a significant level (P. Value <0.05).

Results and discussion

Congenital toxoplasmosis is dangerous disease responsible for almost 1.20 million cases of lifelong disability scattered around the world (21), this imposes an urgent need for the early detection of parasitic infection of women and the study of prevalence and registration of infection rates in residential areas.

Using the ELIZA technique, the results of this study showed that number of positive samples for the examination was 65 /180 samples (36%), the samples that gave a positive result of antibodies IgG 48 (73%) and positive samples of antibodies IgM 22 (33%), this result refers to the infection occur in a pre-pregnancy period, while the rate of modern infection was much less, , this agree with (16) when they are screened for *T. gondii* in the blood samples of (58) aborted women were attend to Al-Batool Maternity Teaching Hospital in Baqubah city/ Diyala province when recorded infection rate (44%), the seroprevelance of IgG (37%) and IgM (5.2%) respectively in non-pregnant women while the anti-toxoplasma antibodies were (38.1%) IgG and (2.4%) IgM in pregnant women, and study 0f (22) when she examined (110) blood samples of pregnant women were collected from special laboratory for Dr. Luay Ibrahim to Pathogenesis Analyses in Baghdad. Iraq Al-Mansour- 14 Ramadan street and showed that the seroprevalence of *T. gondii* IgG (63.4 %) and IgM (13.6 %), our results are consistent with (23) in Babylon when they recorded the presence of IgG antibodies (13.37%) and IgM (6.04%), and study by (24) which recorded a positive IgG infection rate of (24.6%) and IgM (1.34%), we also concur with (25) in Qatar when they studied serology for a total of 823 women of childbearing age and recorded (35.1%) positive IgG infection and (5.2%) IgM antibodies, The difference in the incidence of toxoplasmosis in the above studies may be due to the difference in number of samples, age groups studied, the different methods of serological examination used to examine the blood samples, as well as the difference of society targeted by the study and the different environmental conditions, customs, traditions and health and cultural awareness of the community as most parasitic infections are closely related to the environmental conditions surrounding the individual (26). The extracted DNA samples of the patients were examined using a Real-Time PCR (TaqMan probe) molecular diagnostic technique to detect the B1 gene of the *T. gondii* parasite in the patient's DNA, The results showed the presence of the gene B1 in 28 /180 (15.5%), were figure (1) shows the amplification plot of the Real-Time PCR reaction for product positive samples for the *T. gondii* parasite diagnosis by B1 gene, where the (y-axis) represents the dye measurement unit while the (x-axis) represents the thermal cycles of the reaction:

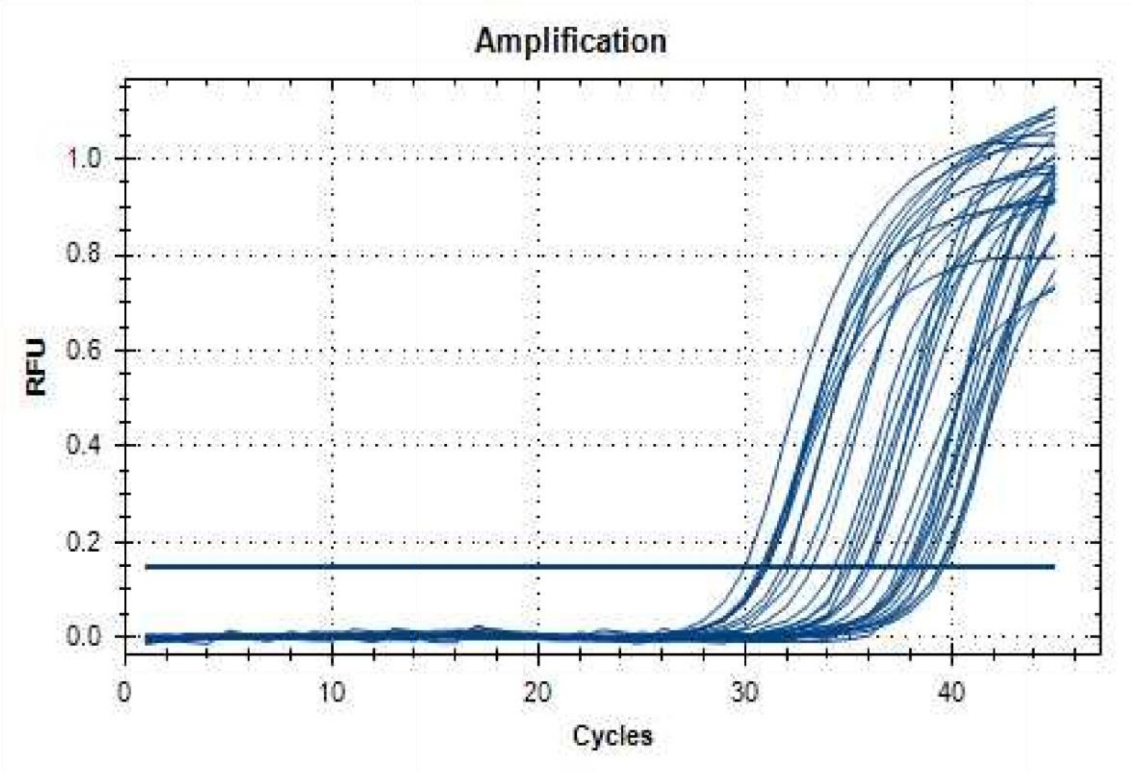

Figure (1): the amplification plot of the Real-Time PCR of B1 gene in *Gondii* from blood

Samples of women under study

This result agree with (16) in in Baqubah city/ Diyala where they recorded an infection rate (15%) by used Real-Time PCR, and with study (27) in Egypt when they used traditional(PCR)polymerase chain reaction to identify for *T. gondii* in aborted women and they recorded (12.5 %), and we agree with (28) when they detected of the parasite in blood samples from aborts woman reviews for Al-Mothana hospital laboratoryand record (16%) by using Real-Time PCR based TaqMan prob. However, our results were higher than recorded (29) a ratio (3.84%) by using quantitative PCR and (6.92%) by using nested PCR when they examined 161 blood samples for aborted women in Mexico, While were lower than (30) when recorded (52.8%) in pregnant women housewives reviewing in to hospitals in Addis Ababa in Ethiopia, and study (31) in Al-Ramady city in Iraq for pregnant women in early months pregnant and showed (41.4%) by using nested PCR.

Molecular diagnosis of toxoplasmosis is the most accurate and sensitive because the ability of this methods to diagnose the modern infection of the parasite before the body begins to form antibodies, while the serological examination depends on the formation of antibodies after several weeks from infection of the host parasite (20), as it is not limited to the examination of blood samples, as is the case most tests, such as serological tests, but includes the examination of all fluids and tissues of different body, where also can investigate the congenital toxoplasmosis through the molecular examination of DNA extracted from the placenta using a special kit as in the study (32) in Ardabil (Iran) when they used placenta nested PCR to diagnose *T. gondii* in 200 aborted women and recorded infection rate (10.5%), As well as the study (33) in France, which relied on the investigation of the parasite based on DNA extracted from amniotic fluid for the fetus and indicated the presence of parasite in 261 samples of 377 (69%), and Early diagnosis and treatment of congenital toxoplasmosis is also thought to reduce the risk of severe fetal lesions (34), the difference in the results of the B technique between the current study and the other studies is due to differences in methods of extraction of DNA and its methods of work and to the amount of parasite in the sample, in addition to this type of sample under study and population density and surrounding environmental conditions (35).

The current study was also concerned with measuring the blood parameters of women with parasite and comparing them with the control group. In the case of red blood cells, the results showed a significant decrease in the concentration of hemoglobin levels in red blood cells in women with toxoplasmosis compared to the control group as shown in Table (2).

Table (2) shows the relationship between red blood cells and hemoglobin concentration in women with *T. gondii* parasites compared with control group.

Blood parameters	Control n=20	Patients n=65
RBCs 10^6 /MM³	5.411 ± 0.049	5.209 ± 0.041
Hb g/dl of blood	12.144 ± 0.212	*9.171 ± 0.760

* *Significant differences at level of significance P< 0.05

This is agree with (36) which observed in his study of hematological tests for blood samples of pregnant women there was a significant reduction in the measurement of Hb concentration, the size of the blood corpuscles and the total number of red blood cells (TRBs, Hb, PVC), as we agree with (37) who observed in his study reduction in the level of hemoglobin (Hb and PVC) when examining blood samples of pregnant women infected with *T. gondii* parasite, this result may be due to a lack of iron level in the blood as a result of this parasite, resulting in a lack of manufacturing hemoglobin in the red blood cells, causing anemia in women infected (38).As for white blood cells, the study of blood parameters showed a significant increase in white blood cells in the blood samples of women infected with parasite compared to the control group and as shown in Table (3).

Table (2) shows the relationship between white blood cells concentration in women with *T. gondii* parasites compared with control group

White blood cell count	Control = 20	Patients n=65
Lymphocyte	25.066±0.080	×52.871±0.102
Monocyte	6.323±0.032	×14.015 ±0.022
Neutrophil	52.113±0.0242	57.62±5.110
TLC ($X10^3/mm^3$)	5.328±0.188	×9.482±0.366
Erythrocyte ($X10^6$)	8.52±0.25	9.12±0.33
Basophil	1.062±0.038	1.052±0.012

×Significant differences at level of significance $P< 0.05$

These results agree with what has been recorded (37) of the increase in the number of white blood cells in people with the disease of toxoplasmosis, as we also agree with (39) in their study that targeted blood parameters of blood samples of the infected gerbils infected laboratory with *T. gondii* parasite have observed a numerical increase in white blood cells in infected individuals compared to the control group, and the same score was recorded by (40) an increase in the number of white blood cells in the blood samples of rats infected with*T. gondii* , the increase in the number of white blood cells may be due to the presence of the parasite that stimulates the natural immune response and acquired in the body of infected women (41), Where the parasite works to stimulate the immune system of the host and is represented by humoral immunity (42).

References:

1- Nelson, K.E.; Williams, C.M. (2014): Infectious disease epidemiology. Third edition Burlington, MA: Jones & Bartlett Learning.

2- Aranda-Lozano, D.F. (2011). Modeling of parasitic diseases with vector of transmission:

Toxoplasmosis and babsiosis bovine. M.Sc. Thesis

Collage de Matem atica Aplicada Universidad Polit ecnica de Valencia. p62.

3- Galvain-Ramirez, M.L; Perez, L.R.; Agar, S.Y; Avila, L.M;Ruíz, A.S; Corella, DB.; Fernandez, B.J and Sanroman, R.T. (2010).Seroepidemiology of Toxoplasmosis in high-school students in the metropolitan area of Guadalajara, Jalisco, Mexico. Sci. Med., 20(1):59-63. 4- Torrey, E.F. and Yolken, R.H. (2013). *Toxoplasma* oocysts as a publichealth problem.Trends Parasitol. 29:380-384.

5- Flegr, J.; Klapilová, K.; Kaňková, S. (2014). Toxoplasmosis can be asexually transmitted infection with serious clinical consequences. Not allroutes of infection

are created equal. Medical hypotheses.83(3):286-289.6

6-Fallahizadeh, S.; Jelowdar, A.; Kazemi, F. and Cheraghian, B. (2018).

Seroprevalence of AntiToxoplasma IgG and IgM among Pregnant Women of Shush

County, Southwest of Iran.

International Journal of Infection: 5 (2); e12192.

7-Wallon, M. and Peyron, F. (2018).Congenital Toxoplasmosis: A Plea for a Neglected

Disease. *Pathogens*, 7(1) pii:E25.DOI:10.3390/pathogens 7010025

www.mdpi.com/journal/pathogens.

8- QM1 Kong, Lu. SH., Tong. QB, Lou. D, Chen. R., Zheng. B., Kumagai, T., Wen, LY. Ohta,

N., Zhou. X.N. (2012). Loop-mediated isothermal amplification (LAMP): early detection of

Toxoplasma gondii infection in mice. Parasitol,Vect; 3(5): 2-7.

9- Singh S, Munawwar A, Rao S, Mehta S, Hazarika NK. (2014). Serologic prevalence of *Toxoplasma gondii* in Indian women of child bearing age and effects of social and environmental factors. PLoS Negl Trop Dis. . 8(3): e2737.

10- Mohammad, M.; Ahmed, S. and Hussain, A. (2012). Seroprevalence of *Toxoplasma gondii* in couples in Ramadi City using enzyme linked immunosorbent assay (ELISA). International Journal of Medicine and Medical Sciences Vol. 4(3), pp. 55-59.

11- Hashoosh, D. and Majeed, I. (2014). Comparison of two assays in the diagnosis of toxoplasmosis: immunological and molecular. East Mediterr Health J. 20:46–50.

12- Robert-GangneuxBelaz. (2016). Molecular diagnosis of toxoplasmosis. Curr Opin Infect Dis.

29:330 – 339. https://doi.org/10.1097/QCO.0000000000000275 PMid: 27191201.

13-Su C., Shwab E.K. and Zhou P. (2010)."Moving towards an integrated approach to molecular detection and identification of Toxoplasma gondii".Parasitol. 137(1): 1-11. . DOI:
10.1017/S0031182009991065.

14- https://en.wikipedia.org/wiki/Basra

15- Firouz ZE, Kaboosi H, Nasiri AF, Tabatabaie SS, Golhasani-K F, Zaboli F. A. (2014).Comparative serological study of toxoplasmosis in pregnant women by CLIA and ELISA methods in Chalus City Iran.*Iran Red Crescent Med J*. 16(4): e15115.

16- Darweesh, N. H.; Hussein, R.A.; Salman, S. T. and Shaker, M. J. (2018). Immunological and Molecular study of *Toxoplasma gondii* from aborted women in Diyala / Iraq. Sci. J. Med. Res., 2(6):75- 82.

17- Abbott Diagnostic Division (2008). Cell-dyn Ruby system OperatorsManual.

18- AL-Taei, N. H. J. (2015). Serological and immunological study to the prevalence of Toxoplasmosis among sewage workers, farm workers and sellers of fruits and vegetables in ThiQar province – Iraq.M.Sc. Thesis College of Sciences – Thi-QarUniversity.P 92.

19- Fernanda. S. G.; Sandra, M. N.; Hilda, F. P.; Solange, M. G. (2010). Toxoplasma gondii: diagnosis of experimental and natural infection in pigeons (Columba livia) by serological, biological and moleculartechniques.Rev. Bras.Parasitol. Vet., Jaboticabal, (19), p. 238-243.

20- Al-Mosawi, R. A.; Al-Aboody, B. A. Al-Temimi, M. B. (2015). Diagnostic study of *Toxoplasma gondii* in students of Thi-Qar university-Iraq by Real-Time PCR. J.Thi-Qar Sci. 5 (2):9-15.

21-Fallahi, S. ;Rosta, i, A ; NourollahpourShiadeh, M ; Behniafar, H. And Paktinat, S. (2018). an updated literature review on maternal-fetal and reproductive disorders of Toxoplasma gondii infection.J Gynecol Obstet Hum Reprod. 47(3):133-140.

22- Al-Rawazq, H. S. A. (2017). Seroprevalence of Immunoglobulin G (IgG) and Immunoglobulin M (IgM) and Risk Factors of Toxoplasmosis for A sample of Pregnant Women in Baghdad. Al-Kindy College Medical Journal, 13(.2): 43-48

23 Abd Al-Hussein, E. F.;Nassir, N. F. and Kadhim, A. (2016). Study of Prevalence and Some Immunological Characteristics of *Toxoplasma gondii* Infections in Pregnant Women. Journal of Babylon University/Pure and Applied Sciences, 24(2):526-533.

24- Salan, Mohammad A. 2011.Determination of Antibodies (IgG, IgM) against *Toxoplasma gondii* in some Iraqi individual by using ELISA technique.

25-Abu-Madi M.A., Behnke J.M., Dabritz H.A. (2010).Toxoplasma gondiiseropositivity and coinfection with TORCH pathogens in high-risk patients from Qatar.American Journal of Tropical Medicine and Hygiene, 82,626–633.

26-Ezatpour B., Mohammad Z., Hessam R., Yadollah P., Mehdi A., Farzad E., Mozhgan A. and Maryam J.G. "Seroprevalence of toxoplasmosis in mentally retarded patients in Iranian rehabilitation centers". J. Parasit. Dis. 2015; 39(1): 13–17

27-. Badr M.S., Samar S.A., Walid S.E., Abd- Allah M.A.E., Hesham M.H. and Ahmed N.M.S.(2016) "Genetic polymorphism of Toxoplasma gondii from recently infected aborted Egyptian women". J. Egypt. Soc. Parasitol. (JESP). 46 (1): 49 - 55.

28- Al-nasrawi, H. A. Naser, H. H. and Kleaf, S. F. (2014). Molecular Detection of *Toxoplasma gondii* in Human and Chicken by Real-Time PCR Technique. International Journal of Advanced Research.2(3); 1023-1027.

29-Hernández-Cortazar, V. B.; Acosta-Viana, K. Y Guzman-Marin, E. ; Segura-Correa, J. C.; Ortega-Pacheco, A.; Carrillo-Martínez, J. R.; González-Losa, M. D. R.; Conde-Ferraez, L. and
Jimenez-Coello, M. (toxoplasma gondii in women with recent abortion from Southern Mexico). Asian Pacific Journal of Tropical Disease 6(3):193-198

30-Gelaye W., Tadesse K. and Asrat H. (2015) "High prevalence of ant toxoplasma antibodies and absence of Toxoplasma gondii infection risk factors among pregnant women attending routine antenatal care in two Hospitals of Addis Ababa, Ethiopia". International Journal of Infectious Diseases 34: 41-45. DOI:10.1016/j.ijid.2015.03.005.

31-Turkey, S. A.; Al –Ani Raja, S. F. and Al-hadithi, J. (2014). Comparison Between the Efficiency of nested PCR Analysis and IgG Avidity Test in Detection Of Acute *Toxoplasma gondii* Infection in Early Pregnancy in Ramadi city.

J. of University of Anbar for pure science: 8(3).

32- Matin S. ;, Shahbazi, G.; Namin, S. T ; Moradpour, R. ; Feizi,F. and Piri-dogahe, H. (2017). Comparison of Placenta PCR and Maternal Serology of Aborted Women for Detection of *Toxoplasma gondii* in Ardabil, Iran.Korean J Parasitol. 55(6): 607–611.

33-Martine Wallon, M.; Franck, J. ; Thulliez, P.; Huissoud, C.; Peyron, F. ; Garcia-Meric, P.; and Kieffer, F. (2010). Accuracy of Real-Time Polymerase Chain Reaction for *Toxoplasma*

gondii in Amniotic Fluid.*ObstetGynecol,115 :(727–733).*

34- . QM1 Kong, Lu. SH; Tong Q.B.; Lou, D.; Chen, R. ;Zheng, B.; Kumagai, T.; Wen, L.Y.; Ohta, N. and Zhou, X.N. (2012). Loop-mediated isothermal amplification (LAMP): early detection of Toxoplasma gondii infection in mice. ParasitVect, 3(5): 2e7.

35- Al-Abodi, H. R. J. (2015). Molecular and Environmental study for species*Entamoeba histolytica, Entamoeba dispar, Entamoebamoshkoviskii* in Al-Qadisiya Province. PhD. Thesis the College of Education, University of Al-Qadisiyah.P. 120.

36- Zakaria, E. G. (2013). Comparative study of hematological changes and therapeuticallyeffect of toxoplasmosis in Nineveh´s women and small ruminants.Bas.j.vet.Res. 12(1).

37-Mahmood, O. I. (2016).Effect of Toxoplasmosis on hematological, biochemical and immunological parameters in pregnant women in Tikrit city, Iraq. Tikrit Journal of Pure Science, 21 (3): 24-27.

38- Javadi, S., Asri Rezaei, S., Tajik, H., Hadian, M., and Shokouhi, F. (2010) Hematological changes of cats with Toxoplasma gondii-specific antibodies'. Comp Clin Pathol. 19:307–310. 39- Atmaca, N.; Çinar, M.; GÜner , B.; Kabakci, R. ; Nuriye, A. ; GazyaĞci, H. T.; Atmaca,, H. T. and Canpolat, S. (2015). Evaluation of oxidative stress, hematological and biochemical parameters during Toxoplasma gondiiinfection in gerbils. Ankara Üniv Vet Fak Derg, 62, 165170.

40- Tonin, A.A ; Silva, A. S. ; Thorstenberg, M. L., Castilhos , L. G. ; França, R.T. ; Rosa Leal, D. B.,; Duarte, M. M. M. F. ; Vogel, F. S. F., La Rue, M.L. and Anjos Lopes, S.T. (2013):Influence of Toxoplasma gondii Acute Infection on Cholinesterase Activities of Wistar Rats.Korean J Parasitol, 51, 421-426.

41-Iwasaki, A. and Medzhitov, R. (2010) Regulation of adaptive immunity by the innate immune system. Science327:291-295.

42- Vigil A, Davies DH, Felgner PL. Defining the humoral immune response to infectious agents using high-density protein microarrays. Future Microbiol. 2010; 5:241–251.

Shipping Technology Based on Quranic Education: An Innovation Courseware

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Nowadays, students more interesting learn using multimedia approach than learning with a book. This courseware is suitable for students who are want to learn more about marine technology in the Quran. The purpose of the courseware is to study about shipping technology in surah Hud. This surah verse 44, explain about ship that use by Prophet Noah to protect himself and his followers from flood. The amazing of the Noah's ark is it can cover many people in the ship. People can know about surah Hud and ship technology in this courseware and know the creation of ship had written in the Quran. Other than that, the method that used in this courseware is historical method to analysis the data with collect verses in the Quran about ships, used method ADDIE concept to develop this courseware. Interface of this courseware is divided into five pages. First page in this courseware is homepage. In this page, introduction about Scientific Interpretation of Quran. In this part explain about definition of Scientific Interpretation according Islamic scholars. Second page is a story about Surah Hud verse 44 according to Islamic scholar. This verse is about Noah's ark is the first ship in the world and explanation the characteristic of Noah's ark. Third page is a video about creation of ship. Forth page is an explanation about ships technology according principles Archimedes and newton on ships or in scientific perspective. Last page, the courseware add the other verses from other surah about ships. The courseware is to make easy for student to learn Scientific Interpretation of Quran using multimedia approach. Using this method is make the courseware is more attractive and more interested. Courseware use a flash to design a courseware. By using flash, the courseware can combine all element of multimedia such as sound, picture, animation, and video easily. It can make the courseware look beautiful and interested for student and people to learn it. The courseware also can help lecture or teacher in learning process with students.

A Decision Tree Model for Software Development Teams

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A model for hybrid use of Activity-based Costing approaches

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Abstract

The Activity-Based Costing (ABC) is an approach to manage costs particularly discussed in the past. A lot of works had as topic this costing system but several authors highlighted as relevant the academic discussion more than the use in the companies. By the point of view of theoretical approach the ABC is an effective tool for defining cost object, resources consumption and so on, but the practical use was limited. The adoption of ABC is complicated and there numerous problems to resolve for its application.

The ABC is revamped in the research field as an important costing system that could resolve the growing welfare costs in the countries more developed. In particular, the healthcare sector needs advanced costing systems and this is increasing over time. Activity-Based Costing and newest model Time-Driven Activity-Based Costing (TDABC) are being more widely adopted with regards to healthcare issues. At the same time, however, we only see a sporadic use of them as they are difficult to use or because they do not provide an overall solution. So by using our own previous empirical study referring to the adoption of ABC in measuring two healthcare processes (at Perugia's Hospital, Umbria Region - Italy) we come across a model for dividing the cost of resources in four levels of analysis.

Using a hybrid ABC system, namely with the combination of the features of traditional ABC and the lighter approach of TDABC, and finally, the traditional cost allocation. So, we build up a matrix that helps us to map and track the costs of resources absorbed in accordance with both variables: the relevance in costs sustained and its variability over time.

The method used is a case analysis and by identifying resource costs of object costs we have singled out these two variables as being important for defining a framework that could improve the chances of being able to adopt the ABC system. We take into consideration the more accurate and more complex analysis of ABC, to the simpler TDABC and traditional allocation, depending on specific resource which is under analysis.

The main result is the increase of the theory on the application of ABC in the healthcare sector and, in particular, how to shift between different activity-based models in order to achieve the information and reduce, on the other hand, the cost of implementing this costing system in a healthcare institution.

We can conclude that this study needs more in-depth analysis because it is based on the only research of its kind, and it therefore requires more detailed case studies to support it, while it does highlight some interesting aspects for future analysis and for discussion on this field of research.

Introduction

In recent years, healthcare changes have created considerable pressure on national welfare, in particular, regarding costs that are sustained for healthcare services. So starting from a project that had its main goal of applying an ABC model to healthcare processes in Perugia's hospital (Umbria Region - Italy), we discover some relevant results in this study because the complexity of healthcare processes means that they can only be managed by an integration of activity-based models.

Review of literature

The application of ABC in the healthcare sector is recent but some studies have been published since the development of the ABC in the 1990s (Chan 1993, Ramsey 1994, Udpa 1996, Baker 1998). In the 2006 Cao et al. introduced a simplified ABC model (defined as SABC by the authors) into a Japanese hospital context with the aim of reducing the complexity of this cost system. Other contributions have been published in recent years but all of them are characterized by their reduced use in specific departments and/or care services (Agyar et al. 2007, Kuchta, Zabek 2011, Goldberg, Kosinski 2011, Rajabi, Dabiri 2012, Hennrikus et al. 2012). In general, many problems to the adoption of the ABC have been detected over time (Tse, Gong 2009). The limited use of the ABC is due to the considerable difficulties in spreading this costing system over an entire healthcare system.

The main problems with its extensive use are: the assessment of performance (also taking into consideration the healthcare results of the patient), the mapping of activities because, in many cases, the healthcare process is not standardized, and finally, the complexity and size of the organization (Popesko, Novak 2011, Popesko, Novak 2013, Gosselin, 1997, Malmi 1997). In recent times, even the Time-Driven ABC (TDABC), introduced in 2007 (Kaplan, Anderson 2007), that was revised for use in healthcare, tries to resolve some application problems (Kaplan 2014, Kaplan, Haas 2014, Kaplan et al. 2014, Kaplan, Porter 2011).

A research question and method

By starting from this issue, we introduce a theoretical framework that could help the application of Activity-based Costing (ABC) to healthcare institutions.

In our contribution, we consider a combined use of ABC, TDABC and traditional allocation depending on the complexity of the healthcare process in question. So our main research question is:

-Is it possible to use a flexible approach in healthcare procedures using an activity-based model and reducing the difficulties of activity mapping?

By using the results of our ABC application in healthcare institutions we build up a matrix that is useful as a guide for cost analysis. In particular, we should consider that in the healthcare process not all resources absorbed have the same significance/relevance and variance in time so, the combined use of ABC and TDABC can performed in relation to this assumption.

In particular, each healthcare process that we analyzed was composed of numerous items cost with each one having a significantly different nature and evolution in time (D'Achille 2017). This aspect, if not considered, could lead to a failure in the adoption of the ABC model due to its complexity, that is greater compared to the industrial sector, for example.

So by the in-depth analysis of the processes, we developed a matrix (as in Figure 1) that could help to address, in simpler fashion, the resources that associates for each one a level of analysis (by analysis we mean tracking, measure, forecasting of costs by using different information media as database, interviews, and so on).

Figure 1

Matrix of deep analysis

		Relevance	
		Low	**High**
Variability in time	**High**	average analysis-	higher analysis
	Low	lower analysis	average analysis +

In this matrix we have four levels referring to the resources. So we therefore have to assume that for each level, a different level of analysis should be associated, the deeper the analysis for resources used that are considered as being expensive and with frequent cost variations over time, while the less detailed analysis applies to resources that are cheaper and/or linked to time variables. The four levels are:

Higher analysis (high relevance and variability): this analysis is particularly deep in this level because it enormously affects the cost object. For example, various high cost medical devices for which the cost can change in the future due to innovation or other external factors that the healthcare institution cannot control.

Average analysis + (high relevance and low variability): the plus symbol identifies the greater importance of cost rather than variability, so in this case the analysis is particularly deep but less so than in the previous level, because cost is relevant while there is some control over it. For example, personnel cost is relevant but the institution can plan and forecast for this according to its needs.

Average analysis - (low relevance and high variability): the minus symbol identifies inexpensive resources but with reduced control over costs for the healthcare institutions. This is the case of common drugs, which are extensively used, but the price of which can change frequently and only one control for the entity is the choice of pharmaceutical company.

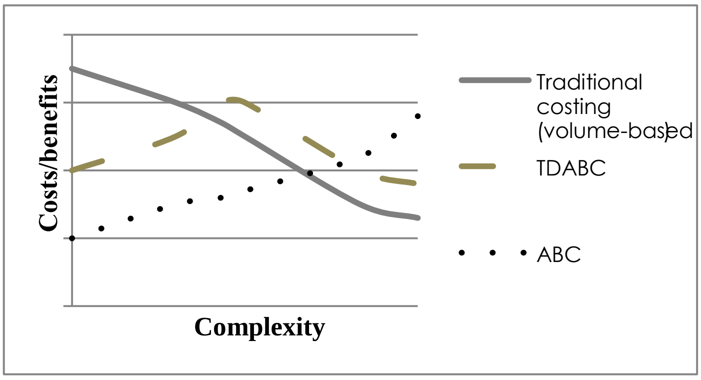
Lower analysis (low relevance and variability): the lower analysis is sufficient in this level so forecasting and extensive use of time-driven approach is correct. In this level, we have resources that are not expensive and that are easier to assess, for example laundry and meats. Also traditional cost allocation, that is volume-based, it is sufficient for this aim.

So in the higher levels the traditional ABC is more effective while in the lower level analysis TDABC provides more benefits. In fact, the TDABC is less expensive to adopt while proving to be useful when the processes are strongly standardized or when there is an important link with time; on the other hand, the ABC is more suitable in the presence of the firm's heterogeneous processes (Siguenza-Gutman et al. 2013). Also the accuracy of the ABC results is higher compared to those of TDABC (Schuhmacher - Burkert 2013) but more expensive to adopt and to up-date in time. Whether different scholars (Levant et al. 2010) find that TDABC is not so useful than ABC and the extensive use could led to Bedaux Point system (Gervais et al. 2009). However, if the attention is only on the process or procedure and it is not used specifically for tracking personnel's time this concern is reduced.

Finally, we can consider that inside of complex process there are resource used that could be allocated sharing by some volume-based parameter. This is the traditional approach to allocate for volumes and it would be applied when the resource is a common cost for more than one process (or cost object) and only if it is respect the requirements of lower analysis, as explained in the matrix.

By a theoretical approach we can highlight a different level of cost/benefits respect to complexity of cost object (as in Figure 2), so we could consider the traditional cost allocation as suitable for processes or part of them that are lower relevance and variability (lower analysis in our matrix) because the cost for implementation is reduced and the information that it provides is well enough. Increasing the complexity of process or part of it, the activitybased models have best performance for achievement valuable information, with the TDABC that is easier to adopt than ABC, but the latter is the main tool for higher complexity process. Every choice in order to adoption a costing system is depending on process complexity but different models can coexist at same time. Finally, the choice is also depending of the balance between cost of adoption and quality of information.

Figure 2



Findings

We consider the hybrid use of the two main models to be the right solution in the application to healthcare institutions (Barret 2007), so we can take into account both the complexity of healthcare processes and the cost-benefits in the simpler adoption of the activity-based costing model, suitable on a case by case basis, namely, they are usable for the same cost object (for example, a process, a patient, a cost department and so on) changing the level of analysis depending on the complexity of the resource considered. So when the economic process is simple in term of relevance and variance the best choices are the traditional cost allocation methods, up to a certain level of complexity is the TDABC that could be considered but when the process is at higher complexity, in this case the ABC provides best information for the management.

Conclusion

The original contribution of this study uses the theory of applying the ABC models to the healthcare sector, the results of which are not pure speculation but rather they are based on previous empirical research. This one highlighted specific challenges of the healthcare process analyzed and the problems of its application, considering the costs of its adoption. As result, the logical solution is to consider a different level of analysis in the four classes depending on the relevance and variability of the resources used.

The weakest point of this study is that it was only based on one study performed for buildingup the matrix of deep analysis, and further investigation is needed. Future empirical studies should track the relation between the adoption of ABC and the complexity of the model choose by healthcare institution and the rate of success in the adoption.

References

Agyar, E., Ersoy, A., Baykara, M., & Ucar, M. (2007). A practical application of Activity Based Costing (ABC) in an urology department. *7th Global Conference on Business & Economics*, 13-14.

Baker, J., & Boyd, G. (1997). Activity-based costing in the operating room at Valley View Hospital. *J Health Care Finance 24*, 1-9.

Cao, P., Toyabe, S., & Akazawa, K. (2006). Development of a practical costing method for hospitals. *Tohoku Journal of Experimental Medicine 208*(3), 213–224.

Chan, Y.L. (1993). Improving hospital cost accounting with activity-based costing. *Health Care Manage Rev 18*, 71-77.

D'Achille, A. (2017). Activity-based Costing Application for Making Decision of Thoracic Surgical Procedures. *Proceedings of 47th International Business Research Conference*, Osaka.

Goldberg, M.J., & Kosinski, L. (2011). Activity-based costing and management in a hospitalbased GI unit. *Clinical Gastroenterology and Hepatology 9*(11), 947–949.

Gervais, M., Levant, Y., & Ducrocq, C. (2009). Le Time Driven Activity Based Costing

(TDABC): “New Wine, or Just New Bottles?”. *La place de la dimension européenne dans la Comptabilité Contrôle Audit, May 2009, France.* 1-31.

Gosselin, M. (1997). The effect of strategy and organizational structure on the adoption and implementation of Activity-Based Costing. *Accounting, Organizations and Society 22*(2), 105-122.

Hennrikus, W.P., Waters, P.M., Bae, D.S., Virk, S.S., & Shah, A.S. (2012). Inside the value revolution at Children's Hospital Boston: time-driven activity-based costing in orthopaedic surgery. *Harvard Orthopaedic Journal 14*, 50–57.

Kaplan, R.S., McLaughlin, N., Michael, A., Burke, Nisheeta, P.S., Douglas, R.N., Kaplan, A.L., Saigal, C., Mahajan, A., & Neil, A.M. (2014). Time-Driven Activity-Based Costing: A Driver for Provider Engagement in Costing Activities and Redesign Initiatives. *Neurosurgical Focus 37*(5).

Kaplan, R.S. (2014). Improving Value with TDABC. *HFM Healthcare Financial Management 68*(6), 76–83.

Kaplan, R.S., & Haas, D.A. (2014). How not to cut health care costs. *Harvard Business Review 64*(11), 116–122.

Kaplan, R.S., & Porter, M.E. (2011). How to Solve the Cost Crisis in Health Care. *Harvard Business Review 89*(9), 47–64.

Kaplan, R.S., & Anderson, S.R. (2007). *Time-Driven Activity-Based Costing: A Simpler and More Powerful Path to Higher Profits*. Boston, Harvard Business School Press.

Kuchta, D., & Zabek, S. (2011). Activity-based costing for health care institution. *8th International Conference on Enterprise Systems, Accounting and Logistics* (8th ICESAL 2011).

Levant, Y., Gervais M., & Ducrocq, C. (2010). Time-Driven Activity-Based Costing (TDABC): an initial appraisal through a longitudinal case study. *Journal of Applied Management Accounting Research 8*(2), 1–20.

Malmi, T. (1997). Towards explaining activity-based costing failure: accounting and control in a decentralized organization. *Management Accounting Research 8*, 459-480.

Popesko, B., & Novak, P. (2013). Activity-Based Costing Methodology as the Tool for Costing in Otorhinolaryngology Department. *Proceedings of the 6th International*

Scientific Conference on Finance and the Performance of Firms in Science, Education, and Practice, 571-579.

Popesko, B., & Novak, P. (2011). Application of ABC Method in Hospital Management. *Proceedings of the 6th IASME/WSEAS International Conference on Economy and Management transformation* (EMT11), 17-19.

Rajabi, A., & Dabiri, A. (2012). Applying Activity Based Costing (ABC) Method to Calculate Cost Price in Hospital and Remedy Services. *Iran J Public Health 41*(4), 100–107.

Ramsey, R. (1994). Activity-based costing for hospitals. *Hospital & Health Services Administration 39*, 385-395.

Schuhmacher, K., & Burkert, M. (2013). Time-Driven ABC: 'Old Wine in New Bottles?' An Experimental Comparison of Time Estimates in Standard ABC and Time-Driven ABC.

AAA 2014 Management Accounting Section (MAS) Meeting Paper.

Tse, M.S.C., & Gong, M.Z. (2009). Recognition of idle resources in Time-Driven ActivityBased Costing and resource consumption accounting models. *Journal of Applied Management Accounting Research 7*(2), 41-54.

Siguenza-Guzman, L., Van den Abbeele, A., Vandewalle, J., Verhaaren, H., & Cattrysse, D. (2013). Recent evolutions in costing systems: a literature review of time-driven activity based-costing. *Review of Business and Economic Literature 58*(1), 34-64.

Udpa, S. (1996). Activity-based costing for hospitals. *Health Care Management Review* 21(3), 83–96.

Long-Term Investors and Market Liquidity: Evidence from EM Pension Funds

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Financial Derivatives' Usage for SME Businesses: The Qualitative Study to Identify the Non-Financial Determinants

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Financial derivatives play vital role in the financial performance of the firms by mitigating different financial risks. It was determined in the literature previously that financial derivatives were being used in larger firms but now financial derivatives are being used in SMEs to accelerate SMEs' financial performance. Several studies had been conducted on larger firms but there was a paucity of literature on the usage of financial derivatives within SMEs. This gap produced motivation for the research in discovering the non-financial determinants of the financial derivatives' usage within SMEs. The research problem: "how and why the non-financial determinants of the usage of the financial derivatives could be established within SME businesses?" Qualitative research approach was used to explore this research question. Purposive snowballing sampling technique was used to identify respondents of this research. Convergent interviews technique was used to collect data in order to develop the literature and to approve the non-financial determinants of the financial derivatives' usage within SMEs. Thematic data analysis technique was used to analyze the collected data. The findings of this research identified total six (6) non-financial determinants of the financial derivatives' usage, i.e. overall risk reduction, level of expertise, risk attitude, high corporate governance, decision making units and awareness. Additionally, this research provided theoretical, methodological, practical and policy implications. This research built a revised theoretical framework, which provided ground for future research.

Keywords: non-financial derivatives, qualitative research, convergent interviews, purpose snowball sampling, SMEs

Institutional environment and new business formation in the EU countries

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Abstract

The formation of new businesses has a positive impact on the growth and development of the European Union economies. Several empirical studies have indicated that supporting entrepreneurship and the creation new businesses, and increasing their role in the development of national economies would not be possible without a solid and effective institutional environment. Therefore, the main objective of our paper is to empirically investigate the impact of the institutional environment-specific factors on the dynamics of the creation of new businesses (as reflected by the level of entrepreneurial activity in its early stage). Our analysis focuses on eighteen European Union member countries (Belgium, Croatia, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, the Netherlands, Portugal, and the United Kingdom) and covers a period of fifteen years, between 2002 and 2016. In our investigation, we express the creation of new businesses through the entrepreneurial activity in its early stages, proxied by the total early-stage entrepreneurial activity rate, the nascent entrepreneurship rate, and the new business ownership rate. We considered these three indicators as dependent variables in our regression models. As independent variables, we included in the analysis several indicators defining the institutional environment of a country, namely: the global competitiveness index, economic freedom, governance quality and the number of procedures needed to start a new business. We have also considered some macroeconomic variables as control variables, namely GDP per capita, unemployment rate and domestic credit to the private sector. The data used for our empirical analysis come from GEM Key Indicators, World Bank DataBank, World Economic Forum, and Heritage Foundation.

The results of our empirical investigation generally are in line with the findings of some

other relevant empirical studies. Thus, we find that the level of economic freedom is positively related to the level of entrepreneurial activity in its early stage. On the other hand, global competitiveness, the governance index and the number of procedures needed to start a new business are negatively associated with the level of entrepreneurial lactivity. Our main conclusion is that the quality and soundness of the institutional environment in a country are of major importance for encouraging the creation of new businesses and the development of entrepreneurial activity. Therefore, our results could serve policymakers in their quest to strengthen the institutional environment, so that it can support the development of entrepreneurship.

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Macroeconomic Environment and Shadow Banking Development in Central and Eastern Europe

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Abstract

Shadow banking – either in its narrow approach, or in the broader one – has become more significant in the last years, especially during and after the recent financial crisis of 2007/2008. Our study aims to quantitatively assess some of the determinants of shadow banking dynamics in eleven EU countries from Central and Eastern Europe over the period 2004-2017. Given the fact that there are two approaches of tackling shadow banking institutions, the current study takes into consideration both of them, namely the broad one (including all non-monetary financial institutions, except insurance corporations and pension funds) and the narrow one (excluding from the above one the non-MMF investment funds). Thus, by employing panel data estimation techniques, we alternatively assess the impact of six important macroeconomic and financial variables on two dependent variables pertaining to the two different definitions. The analysis is conducted on quarterly data coming from three main publicly available data sources. Our findings confirm that shadow banking is sensitive to overall macroeconomic conditions, economic growth positively influencing the expansion of this segment of the financial sector. In addition, a higher demand of funds from institutional investors, which also reveals a more developed financial system, supports the expansion of the shadow banking sector. Moreover, in a low interest rate environment the search for yield makes investors turn to shadow banks, while the development of the shadow banking sector is also found to be complementary to the development of the rest of the financial system, in particular traditional banks.

Keywords: shadow banking; macroeconomic and financial determinants; panel data estimation techniques; Central and Eastern Europe

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AN ASSESSMENT OF HOSPITAL SUPPLY CHAIN EFFICIENCY IN NORTH CENTRAL NIGERIA

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Improving hospital supply chain efficiency in a developing country such as Nigeria has become increasingly important as health care organizations all over the world are striving to improve operational efficiency as well as reduce cost. This study therefore assessed the efficiency levels of the decision-making units of public hospitals in using their supply chain towards meeting the satisfaction of patients and health workers. To assess the perceived supply chain efficiency from the perspective of hospital staff and patients, data was collected from hospital staff and patients in Jos, Plateau State - Nigeria using two sets of structured questionnaires while T-test was used to test the stated hypothesis. Findings revealed that while treatment was satisfactory, the coordination between units was slow, the waiting time long and feedback almost non-existent. This was significantly caused by the persistent difficulty in streamlining the supply chains. Based on these findings, insights were offered on the important efficiency dimensions that Nigerian hospitals should concentrate on in order to meet staff needs and serve their patients effectively.

Key word: Efficiency, Assessment, Supply-chain, Nigeria

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